This book presents solutions to the problems appearing at the end of each chapter in the third edition of *Decision Analysis for the Professional* (titled *Decision Analysis with Supertree* in the second edition). As will be noted throughout this book, many different solutions are possible for some of the more complex problems and case studies. For these problems, the answers depend on how the student chooses to structure the problem. For instance, answers may differ on how the variables are defined, what value model is used, the structure of the tree, and the like. To focus class discussion and keep answer methods consistent, instructors may want to review the possible solutions presented here before assigning problems in order to decide if any specific guidelines should be given to students.

The solutions presented make use of the Supertree and Sensitivity software (version 10.31) as much as possible. Other decision tree modeling packages could be used as well. Spreadsheet models are also used where relevant. In this book, spreadsheet models are done in Excel, though any of the packages Supertree is capable of interfacing with will do. Although the software is not needed for many of the basic problems (indeed, most of the basic problems do not use it), the solutions for the more sophisticated problems become quite laborious without software. Instructors will have to decide on the balance between requiring calculations done by hand (a necessary part of the learning process) and calculations done by computer (one of the important tools needed for practical decision analysis.)

Instructors may find that the solutions use unfamiliar shortcuts and representations to analyze the problems. These shortcuts and “tricks of the trade” will be of particular interest to professional decision facilitators (and to future professional facilitators) because they simplify the analysis and keep the computer representation of the problem manageable. The combinatorial nature of the tree makes it all too easy to exceed the limits of any computer system.
Finally, we have corrected several errors in the statement and solution of problems. “We learn from Horace, ‘Homer sometimes sleeps.’”¹ There are undoubtedly further errors in some of the solutions, but we hope none serious enough to defy a reworking. We would appreciate being informed of any errors, or of any suggestions for improving the solutions. It would be a pleasant task to meet demands for a fourth edition.

John Celona
Peter McNamee
Menlo Park, California

¹Byron said it—did he also know how many mistakes we would make while still awake?
Contents

Chapter 1—Introduction 1
Chapter 2 — Uncertainty and Probability 5
Chapter 3 — Decisions Under Uncertainty 25
Chapter 4 — Probabilistic Dependence 71
Chapter 5 — Attitudes Toward Risk Taking 87
Chapter 6 — The Complexity of Real-World Problems 115
Chapter 7 — Corporate Applications of Decision Analysis 135
Chapter 8— Corporate Decision Making 157
Chapter 9 — Decision Quality 179
Chapter 10 — Probability Theory 183
Chapter 11 — Influence Diagram Theory 209
Chapter 12 — Encoding a Probability Distribution 223
Problems and Discussion Topics

1.1 Describe the difference between good decisions and good outcomes.

Answers for this question should discuss the difference between a decision (an allocation of resources under your control) and an outcome (one of several possible events that occurs for a given uncertainty, which, by definition, is not under your control).

Good decisions, then, occur when resources are allocated in a logical manner consistent with the decision maker’s values and information. One of the tasks in achieving this consistency is balancing the cost of analysis against the stakes involved—small stakes normally do not justify undertaking large analyses.

Good outcomes are a matter of chance (and luck?). You can hope to enjoy good outcomes, but you can only insure that you make good decisions.

1.2 Describe your own approach to making important decisions. Do you use a systematic approach in making them? Do you try to make decisions in a consistent manner? Have you been satisfied with the major decisions you’ve made so far (or just happy or unhappy with the outcomes)?

Students should, at least, discuss weighing different alternatives in making a decision. They might, for instance, describe just “thinking about” the alternatives until they decided which one they liked best, or they may be more detailed and, for instance, discuss weighing conflicting goals, or even comparing different costs and benefits. The purpose of the problem is to get them thinking about the decision bases for some decisions they have made (values, information, alternatives) and the logic (or non-logic) applied to that basis to arrive at a decision.

In examining whether they were happy or unhappy with past decisions, students should consider whether they were happy with the decision itself (did they choose the best alternative under the circumstances) or whether
they were lucky enough to enjoy a good outcome, or both.

Another possibility is to ask them to consider a decision for which the final outcome is not yet certain (such as the decision to pick a particular major in college). Students might consider whether they will be satisfied with the decision made regardless of the outcome (they could not have made a better choice), or whether they are (perhaps anxiously) awaiting the outcome.

1.3 How did you make your decision on which college to attend? Does hindsight reveal any shortcomings in the decision process?

The considerations in this question are similar to those of the previous questions. Instructors might help students realize the potential complexity of the choice of college by bringing out such possible considerations as: overall reputation, quality of particular programs of interest, tuition costs, extracurricular and athletic programs, living facilities and costs, distance from home, presence or absence of friends there, climate and location, student/faculty ratio, availability of guacamole or hoagies, etc.

1.4 What concerns would you like a decision-making methodology to address?

In answering this question, students should think about addressing early in the process the trade-offs between different, possibly competing values (e.g., time, money, fondness for sports cars, dislike of uncertainty and risk) to make the decision-maker comfortable with the trade-off finally chosen; addressing complex situations where confusing or involved circumstances may not make the actual trade-offs apparent; and dealing with uncertainties that may determine which values are actually best satisfied.

Other possible considerations may include having the methodology (or methodologies) be flexible enough to keep costs commensurate with the stakes in the decision or knowing when to apply particular methodologies to particular problems (e.g., decision analysis, linear programming, talking to your spouse, etc.). Finally, a good methodology should tell the decision-maker when to terminate the process and proceed to action.

1.5 Describe a decision you currently face in which uncertainty is an important factor. Will you find out the outcome of the uncertainty before or after you make your decision?

This question is intended to test whether students understand how to identify uncertainties relevant to a particular decision and to get students thinking in terms of when an uncertainty will be resolved relative to when the decision needs to be made. Generally, an uncertainty is relevant to a decision if it will affect the final outcome enjoyed. It is important to determine if an uncertainty is resolved before or after a decision because the alternative chosen may be different in the two cases. This timing also becomes relevant later in the process when we change the order of uncertainties and decisions and calculate the value of information.
1.6 Can an uncertainty be an important factor in a decision when the outcome may never be discovered? Describe why or why not and give an example.

Because decisions eliminate possible future states of the world, there are many situations in which uncertainty is an important factor in decision-making but the outcome is never discovered.

Consider, for example, the decision whether to market a new product with a highly uncertain market potential. If the product is not introduced to the market, perhaps because of this uncertainty, the actual market potential will never be known.

In litigation, a settlement means that the uncertainty concerning a jury’s verdict will never be resolved.

The choice of a career path or marital state or place of residence means that the uncertainties surrounding paths not chosen will never be resolved.
2

Uncertainty and Probability

Problems and Discussion Topics

2.1 Consider today’s weather forecast. Are the chances of rain or sunshine expressed verbally or with probabilities? What is your probability of rain given that weather forecast? If your probability is different from the forecast, does the difference stem from your and the forecaster’s different states of knowledge or from some other reason?

Answers to this question should identify a “20 percent chance of rain” as a probability, or “very likely” as a verbal expression of probability. In comparing their own probability to that given in the newspaper, students might attribute any difference to a different state of knowledge (the weather forecaster presumably looks at satellite photos, National Weather Service forecasts, and the like) or they might question the estimating process used by a particular forecaster (e.g., the forecaster does not think about probability the way it is used in this book). However, most forecasters are rather sophisticated in dealing with probability and information and have a better state of information than the casual observer. A student disagreeing with today’s forecast might consider whether he or she has systematically evaluated the past accuracy of forecasts.

2.2 In the section “What Are Probabilities?” are statements like, “Probabilities represent our state of knowledge.” Such statements are sometimes misinterpreted to mean that probabilities are arbitrary numbers between 0 and 1. In fact, probability is a well-defined concept with very strong implications. For example, if two people have exactly the same information (knowledge) about an event, they should assign the same probability to this event. Furthermore, if new relevant information becomes available, the prior probability assignment will have to be changed (updated).
What else can you infer from the statement “Probabilities represent our state of knowledge”?

A number of responses to this question are possible, depending on how students consider their state of knowledge and numerical representations of it. A basic observation is that any new information making an event seem less likely should lower your probability, while information making an event seem more likely should increase your probability. A probability of .5 shows strong information about the symmetry of a problem: either outcome is equally likely. Similarly, a probability of 0 or 1 would represent perfect information and no uncertainty—a situation not normally attained by mere mortals.

Finally, students might consider that a probability can only be as good as one’s state of information. Consistent, coherent probabilities do not result from a state of knowledge with unresolved, conflicting facts or beliefs—e.g., “Marketing forecasts that sales will be up 9 percent next year, but I do not see how we could possibly do better than 4 percent.” Before meaningful probabilities can be assessed, the person must evaluate the apparent contradictions and decide how to incorporate the information into his own knowledge and belief.

2.3 Why is an influence diagram (or similar method) necessary for understanding complex uncertainties? How do the procedure and graphical form of an influence diagram deal with the problem?

In dealing with complex uncertainties, it is necessary to think systematically about many different factors. The mind can only juggle about seven facts simultaneously; as the number of factors grows, some external graphical representation of the problem becomes extremely helpful, if not necessary. In addition, having things on paper helps communication, an especially important consideration in complex problems.

It is important to have a representation that focuses almost entirely on uncertainty. Many people are uncomfortable when dealing with uncertainty, and you will find that discussion is quickly diverted to questions of fact or relationship unless there is a mechanism like the influence diagram to maintain the focus on uncertainty.

Finally, probability trees are adequate for understanding simple sets of uncertainties. As the complexity of the problem grows, however, the probability tree grows to enormous size and becomes an ineffective means for representing the problem. It is at this level that the influence diagram shows itself to be a compact, intuitive, and powerful representation of uncertainty.

2.4 What is the relationship between each component of an influence diagram (arrows, ovals, and double ovals) and the components of a probability tree?

Ovals in the influence diagram correspond to chance nodes in the probability tree. Arrows between two nodes in the influence diagram correspond to probabilistic dependence—the probabilities (or outcomes) at the node at the head of the arrow depend on the branch of the influencing node (the node at
the base of the arrow). Double-ovals are usually not shown in the tree because they represent calculations, not uncertainties. One double-oval node, however, usually furnishes the value shown at the end of each path through the tree; this double-oval is the endpoint node in Supertree.

2.5 Can you draw a probability tree directly without first drawing an influence diagram? When would this be a bad or good idea? Does your answer depend on the level of expertise of the person doing the analysis?

Probability trees and influence diagrams are representations of the same problem, and the analyst can use either or both representations in the course of analyzing a problem. Probability trees are a useful way to start structuring a problem with three or four principal uncertainties and a natural temporal or causal sequence—“If this happens, then that would probably happen...” The Positronics case analysis could easily have started with a probability tree. However, if the number of uncertainties is over five, the compactness of the influence diagram makes it a better starting point.

The expertise of the analyst can be a crucial factor in the choice of method. A really expert analyst can help people realize early that only a few crucial uncertainties contribute to a problem and that either an influence diagram or a probability tree formulation may be appropriate. A less expert analyst may start with a large influence diagram which only later will be simplified to the level at which a probability tree representation is practical.

2.6 In the section “Using Intuition Effectively,” we discussed how to define uncertainty clearly, and the role of the clairvoyant in the clairvoyance test. How can the clairvoyant help you? Can the clairvoyant change your future?

The main point here is that the clairvoyant is a character who does not deal well with ambiguity. If he can not clearly answer a question with a yes, a no, or a number, then he will ask for more details (“Do you mean gross sales or net sales? Before or after sales taxes?”). Thus, the clairvoyant forces a definition of the event specific enough that a simple yes, no, or number will describe which event will occur. A corollary is that when more specificity is required, the “event” described will often actually be a number of possible events. Of course, almost any event can be further broken down into component events. The level of specificity required is determined by the context in which the event is considered.

The clairvoyant cannot change the future. He can only predict which event will actually occur. To change probabilities and change the future, look for the wizard in Chapter 3.

2.7 What do you do when an uncertainty fails the clairvoyance test? How might this change your influence diagram? How might this change the structure of your probability tree?

As mentioned in the solution to problem 2.6, when an event fails the clairvoyance test, it must either be better defined or broken down into
component events. Usually the clairvoyant only needs a better definition. Sometimes, however, the best way to do this is to split the event into separate conjunctive uncertainties. For instance, if you ask “Will Joe Montana win for the 49ers on Sunday?”, the clairvoyant may respond “Does Joe have to play the whole game to be considered responsible for the win or loss?” As illustrated below, there are two ways to incorporate this distinction into an influence diagram or probability tree.

The second method (breaking the problem into two uncertainty nodes) is usually preferred because people are seldom adept at estimating joint probabilities.

2.8 The expected value is sometimes described as the mean value you would expect to achieve if you undertook the same venture many times. Unfortunately, since many decisions are one-of-a-kind decisions, there is no opportunity to repeat them and establish a historical mean. Suppose, though, that a venture had just been resolved (all the uncertain events had happened) and you were now faced with an identical one. Would your prospective expected value for the second venture be the same as it was for the first? Why or why not?

The answer to this question would depend on whether knowing the resolution of the uncertainties changed your assessment of the likelihood of the same resolution occurring in the future. For instance, if an uncertainty involved achieving a certain yield from a chemical process, then knowing it had been achieved in the past should make doing so again quite likely. In contrast,
knowing that a coin flip turned up tails usually makes no difference in estimates for the next flip; or knowing rainfall was above average in a given year might make no difference in future estimates of rainfall. The difference is in whether knowing one resolution of the uncertainty gives you any insight into the underlying process. If the answer is yes, your information has changed, your probabilities should change, and your expected value will change. If the answer is no, your information, probabilities, and expected value should remain the same.

2.9 You are going to the movies tonight with a new date. You plan on treating, but your date may want to go Dutch treat (each person pays) or treat you. You figure the three outcomes are equally likely. The cost for the movie is $5 per person. You plan to at least buy popcorn if your date wants it (with a 4 out of 5 chance that he or she will). However, you have forgotten how much the large popcorn costs. You would give 5 out of 10 that it costs $2, and split the rest of the probability between $1.50 and $2.50.

You just discovered that you only have $10 cash right now. What is the expected cost of going to the movie tonight? What is the probability that it will cost you more than $10? What is the probability that it will not cost you anything?

Note that in the Supertree input information shown below, the popcorn yes/no decision has been input as 1/0 to allow easy use of the node name in the endpoint node calculation.

<table>
<thead>
<tr>
<th>Tree name: Popcorn</th>
</tr>
</thead>
<tbody>
<tr>
<td>STRUCTURE</td>
</tr>
<tr>
<td>1C2 2 2</td>
</tr>
<tr>
<td>2C3 3</td>
</tr>
<tr>
<td>3C4 4 4</td>
</tr>
<tr>
<td>4E</td>
</tr>
</tbody>
</table>

The tree below shows the rollback to reach the expected cost of attending the movie. Note how Supertree converted the 1/3 probabilities into decimals (only 3 decimal places are shown, but they really do add up to 1.0).
The desired probabilities can be read from the cumulative plot below. The probability of costing more than $10 is one minus the probability of costing less than or equal to $10, or $1 - .70 = .30$. Because zero is the lowest possible outcome, the probability of costing zero is the same as the probability of costing less than or equal to zero, or .05.
2.10 Your prize Rapid Ripe tomato plant has flowered and is ready to start producing fruit. If all goes well, your plant will produce tomatoes by the end of the week. It will then produce a new set of flowers and blossoms next week. Unfortunately, your area is subject to blossom wilt, which causes the tomato flowers to fall off. If the blossoms fall off, a new set of blossoms will not emerge until next week, and tomatoes will not be ready until the end of that week.

Luckily, each time blossoms fall off, the plant builds up its resistance; the probability of each succeeding blossom falling off is then only half as much. You estimate that the probability of this first set of blossoms falling off is .40.

a. Draw the influence diagram for this problem and then draw the probability tree.

![Probability Tree Diagram]

---

**Tomatoes in Week 1**
- Yes: 0.6
- No: 0.4

**Tomatoes in Week 2**
- Yes: 0.8
- No: 0.2

**Tomatoes in Week 3**
- Yes: 0.9
- No: 0.1
**b. What is the probability of tomatoes being ready in the third week?**

The key to evaluating this problem is recognizing that everything works on a one-week cycle, and that the probability of having tomatoes in a particular week is one minus the probability of blossoms falling off that week. Thus, the tree below has chance nodes for tomatoes in weeks one, two, and three, with a zero or one for tomatoes or no tomatoes (so the number of weeks of tomatoes can be simply added up in the endpoint), and the probability of having tomatoes in any one week depends on all the results of all the weeks that came before.

```
Tree name: Tomatoes

<table>
<thead>
<tr>
<th>STRUCTURE</th>
<th>NAMES</th>
<th>OUTCOMES</th>
<th>PROBABILITIES</th>
</tr>
</thead>
<tbody>
<tr>
<td>1C2 2</td>
<td>T1</td>
<td>1 0</td>
<td>.6  .4</td>
</tr>
<tr>
<td>2C3 3</td>
<td>T2</td>
<td>1 0</td>
<td>Depends on 1</td>
</tr>
<tr>
<td>3C6 6</td>
<td>T3</td>
<td>1 0</td>
<td>Depends on 1 2</td>
</tr>
<tr>
<td>6E</td>
<td>B$T1+T2+T3</td>
<td>Depends on 1 2 3</td>
<td></td>
</tr>
</tbody>
</table>
```

The probability, then, of tomatoes being ready in the third week can be obtained by bringing the T3 node to the front of the tree and drawing it. Alternatively, this probability could be obtained by obtaining the joint probabilities and summing the probabilities for those scenarios that have tomatoes in the third week. The resulting probability by either method is .736.

```
Probs T3  Exp Val
.736 1  2.20
.264 0  1.52
```

**c. What is the expected number of weeks you will have tomatoes over the next three weeks?**

Since the endpoint of the tree summed up the number of weeks of tomatoes in each scenario, this number can be obtained by computing the expected value of the entire tree. This number is 2.02, and the full tree is shown below.
d. What is the probability you will lose the blossoms in one or more weeks of the next five weeks?

This number is one minus the probability of no blossoms ever falling off, or

\[ 1 - (.6 \times .6 \times .6 \times .6 \times .6) = 1 - .0778 = .9222 \]

2.11 You have discovered the lights on Van Ness Avenue in San Francisco are synchronized, so your chances of getting a green light at the next intersection are higher if you have not been stopped by a red light at the last intersection. You estimate that there is a 4/5 chance of getting a green light if the previous light was green. Similarly, there’s only a 1/4 chance of getting a green light if the last light was red. Because Van Ness is a major throughway, you estimate there’s a 2/3 chance of the first light being green.

a. Draw the influence diagram for this problem and then draw the probability tree.
b. What is the probability that the second light will be green?

Again, this tree uses outcomes of 0 to represent red lights and 1 to represent green lights. The endpoint simply adds up the number of green lights. The probabilities of the second light being green depend on what the first one was, and the probabilities for the third light depend on the second.
Accordingly, the unconditional probability of the second light being green can be obtained by bringing the second node to the front of the tree and drawing it. This probability could also be obtained by drawing the tree for the first two lights and reversing it by hand. The probability that the second light is green is .619.

<table>
<thead>
<tr>
<th>Probs Green2 Exp Val</th>
</tr>
</thead>
<tbody>
<tr>
<td>382 0 0.60</td>
</tr>
<tr>
<td>619 1 2.67</td>
</tr>
</tbody>
</table>

c. Out of the first three lights, how many lights can you expect to be green? Because the endpoint adds up green lights, this number is the expected value of the tree. This is the expected value of 1.88 shown above in 2.11b (because changing the order of chance nodes does not affect the expected value). The full tree is shown below.

\[
\begin{array}{c|c|c}
\text{Probs Green1 Exp Val} & \text{Probs Green2 Exp Val} & \text{Probs Green3 Exp Val} \\
750 0 0.25 & -750 0 0.00 & -750 0 0.00 \\
250 1 1.80 & 250 1 1.00 & 250 1 1.00 \\
800 1 2.00 & 800 1 2.00 & 800 1 2.00 \\
750 0 0.25 & 750 0 1.00 & 750 0 1.00 \\
200 0 1.25 & 200 0 2.00 & 200 0 2.00 \\
800 1 2.80 & 800 1 3.00 & 800 1 3.00 \\
670 1 2.49 & 670 1 2.49 & 670 1 2.49 \\
330 0 0.64 & 330 0 0.64 & 330 0 0.64 \\
670 1 2.49 & 670 1 2.49 & 670 1 2.49 \\
330 0 0.64 & 330 0 0.64 & 330 0 0.64 \\
670 1 2.49 & 670 1 2.49 & 670 1 2.49 \\
\end{array}
\]

d. What is the probability of the third light being green when the first one was red? (Hint: draw a tree with three nodes representing the first three lights.)

This number can be obtained by obtaining the joint probabilities and then summing the probabilities for those cases where the first light was red and the third one was green. It can also be obtained by drawing the tree with the third node put after the first, as below. In any case, the answer is .388.

\[
\begin{array}{c|c|c}
\text{Probs Green1 Exp Val} & \text{Probs Green3 Exp Val} \\
330 0 0.64 & 612 0 0.08 \\
670 1 2.49 & 690 1 2.93 \\
330 0 0.64 & 388 1 1.52 \\
670 1 2.49 & 690 1 2.93 \\
\end{array}
\]

2.12 An excess probability distribution plots the probability that the value is greater than a given number. Plot the excess probability distribution for the initial Positronics tree (Figure 2–9). What is the probability the value is greater than $130,000?

Since the excess probability plot is the complement of the cumulative probability plot, the graph here should look like the cumulative plot in Figure 2-15 turned upside down. For instance, where the cumulative plot shows that the probability the value is less than $130,000 is .85, the excess probability plot should show that the probability the value is greater than $130,000 is .15.
2.13 Plot the probability mass function for the initial Positronics tree (Figure 2–9). What is the difference between the mass function and the histogram? (The probability mass function is defined in Chapter 10.)

The probability mass function is simply a vertical bar for each outcome. The height of the bar is the probability of the outcome; the bar is drawn at the value of the outcome, as shown below.
The difference between the mass function and the histogram is that the mass function plots each value and its probability directly, while the histogram shows the probability of groups of values as grouped into specific ranges (bins). Thus, the mass function shows the probability of a point, while the histogram shows the probability of a range of values. The histogram becomes more useful for plots and interpretation as the number of values increases, while the mass function becomes progressively more difficult to plot and interpret.

2.14 Suppose the closing trading price for platinum on the world markets today was $550 per ounce. (Does this pass the clairvoyant test?) Toward the end of the day, you put in an order to your broker to purchase one, two, or three contracts to sell 100 ounces of platinum one year from now, depending on how many contracts were available. Given the low volume in platinum contracts recently and given how late you called in, you figure there is about a .3 chance you got one contract, a .6 chance you got two, and a .1 chance that you got three.

You decide to seek out further information on the future of platinum prices. A very nervous metals broker gave you the following distribution on the closing trading price one year from now.

a. Draw the influence diagram for this problem. Is there any information not reflected in the influence diagram, and, if so, how does the influence diagram relate to it?
The influence diagram for the problem as shown below is quite simple.

Even though it is not explicitly shown, we know that each uncertainty node will contain a definition of the possible events (outcomes) and the probabilities of their occurring—the distribution tree. The double-oval node Profit contains a means of calculating the profit for each of the combinations of outcomes of the two uncertainty nodes. Today's closing price of $550 does not appear explicitly in the diagram; it is implicit in the Profit node.

Is there information not reflected in the influence diagram? This question is intended to stimulate discussion and can be answered in several ways. One answer is that all the information in the problem is contained in the diagram (as discussed above), although much of it is implicit in the definition of the nodes. Another possible answer is that the diagram explicitly represents only relationships between uncertainties. The first answer is the most useful way of thinking about influence diagrams.
b. **Discretize the distribution above into a chance node whose branches have probabilities of .25, .50, and .25. What is the expected price of platinum one year from now?**

Discretizing the distribution (below) produces about a .25 probability of $400, a .5 probability of $540, and a .25 probability of 620, yielding an expected price of approximately $525. Answers to this question will have small variations from students' differences in estimating the equal areas and in reading the scale, but should be reasonably close to this answer.

![Trading Price of Platinum One Year from Today ($ per ounce)](chart.png)

\[ \text{Cumulative Probability} \]

\[ \text{Trading Price of Platinum One Year from Today ($ per ounce)} \]

\[ 1.0 \]

\[ 0 \]

\[ 100 \]

\[ 300 \]

\[ 500 \]

\[ 700 \]

\[ 900 \]

\[ 0 \]

\[ 0.50 \]

\[ 1.0 \]

\[ .3 \]

\[ .6 \]

\[ .1 \]

\[ .25 \]

\[ .50 \]

\[ .25 \]

\[ .25 \]

\[ .50 \]

\[ .25 \]

\[ .25 \]

\[ .50 \]

\[ .25 \]

\[ .25 \]

\[ .50 \]

\[ .25 \]

\[ .25 \]

\[ .50 \]

\[ .25 \]

\[ .25 \]

\[ .50 \]

\[ .25 \]

\[ .25 \]

\[ .50 \]

\[ .25 \]

\[ .25 \]

\[ .50 \]

\[ .25 \]

\[ .25 \]

\[ .50 \]

\[ .25 \]

\[ .25 \]

\[ .50 \]

\[ .25 \]

\[ .25 \]

\[ .50 \]

\[ .25 \]

\[ .25 \]

\[ .50 \]

\[ .25 \]

\[ .25 \]

\[ .50 \]

\[ .25 \]

\[ .25 \]

\[ .50 \]

\[ .25 \]

\[ .25 \]

\[ .50 \]

\[ .25 \]

\[ .25 \]

\[ .50 \]

\[ .25 \]

\[ .25 \]

\[ .50 \]

\[ .25 \]

\[ .25 \]

\[ .50 \]

\[ .25 \]

\[ .25 \]

\[ .50 \]

\[ .25 \]

\[ .25 \]

\[ .50 \]

\[ .25 \]

\[ .25 \]

\[ .50 \]

\[ .25 \]

\[ .25 \]

\[ .50 \]

\[ .25 \]

\[ .25 \]

\[ .50 \]

\[ .25 \]

\[ .25 \]

\[ .50 \]

\[ .25 \]

\[ .25 \]

\[ .50 \]

\[ .25 \]

\[ .25 \]

\[ .50 \]

\[ .25 \]

\[ .25 \]

\[ .50 \]

\[ .25 \]

\[ .25 \]

\[ .50 \]

\[ .25 \]

\[ .25 \]

\[ .50 \]

\[ .25 \]

\[ .25 \]

\[ .50 \]

\[ .25 \]

\[ .25 \]

\[ .50 \]

\[ .25 \]

\[ .25 \]

\[ .50 \]

\[ .25 \]

\[ .25 \]

\[ .50 \]

\[ .25 \]

\[ .25 \]

\[ .50 \]

\[ .25 \]

\[ .25 \]

\[ .50 \]

\[ .25 \]

\[ .25 \]

\[ .50 \]

\[ .25 \]

\[ .25 \]

\[ .50 \]

\[ .25 \]

\[ .25 \]

\[ .50 \]

\[ .25 \]

\[ .25 \]

\[ .50 \]

\[ .25 \]

\[ .25 \]

\[ .50 \]

\[ .25 \]

\[ .25 \]

\[ .50 \]

\[ .25 \]

\[ .25 \]

\[ .50 \]

\[ .25 \]

\[ .25 \]

\[ .50 \]

\[ .25 \]

\[ .25 \]

\[ .50 \]

\[ .25 \]

\[ .25 \]

\[ .50 \]

\[ .25 \]

\[ .25 \]

\[ .50 \]

\[ .25 \]

\[ .25 \]

\[ .50 \]

\[ .25 \]

\[ .25 \]

\[ .50 \]

\[ .25 \]

\[ .25 \]

\[ .50 \]

\[ .25 \]

\[ .25 \]

\[ .50 \]

\[ .25 \]

\[ .25 \]

\[ .50 \]

\[ .25 \]

\[ .25 \]

\[ .50 \]

\[ .25 \]

\[ .25 \]

\[ .50 \]

\[ .25 \]

\[ .25 \]

\[ .50 \]

\[ .25 \]

\[ .25 \]

\[ .50 \]

\[ .25 \]

\[ .25 \]

\[ .50 \]

\[ .25 \]

\[ .25 \]

\[ .50 \]

\[ .25 \]

\[ .25 \]

\[ .50 \]

\[ .25 \]

\[ .25 \]

\[ .50 \]

\[ .25 \]

\[ .25 \]

\[ .50 \]

\[ .25 \]

\[ .25 \]

\[ .50 \]

\[ .25 \]

\[ .25 \]

\[ .50 \]

\[ .25 \]

\[ .25 \]

\[ .50 \]

\[ .25 \]

\[ .25 \]

\[ .50 \]

\[ .25 \]

\[ .25 \]

\[ .50 \]

\[ .25 \]

\[ .25 \]

\[ .50 \]

\[ .25 \]

\[ .25 \]

\[ .50 \]

\[ .25 \]

\[ .25 \]

\[ .50 \]

\[ .25 \]
The full tree (below) shows the potential profitability of different scenarios, leading to the overall expected profit of $4.5 million.

<table>
<thead>
<tr>
<th>Probs ContractsExp Val</th>
<th>Probs PriceExp Val</th>
</tr>
</thead>
<tbody>
<tr>
<td>.250 400 15000</td>
<td>.250 400 15000</td>
</tr>
<tr>
<td>.500 540 1000</td>
<td>.500 540 1000</td>
</tr>
<tr>
<td>.250 620 -7000</td>
<td>.250 620 -7000</td>
</tr>
<tr>
<td>.300 1 2500</td>
<td>.250 620 -14000</td>
</tr>
<tr>
<td>.600 2 5000</td>
<td>.250 620 -21000</td>
</tr>
<tr>
<td>.100 3 7500</td>
<td>.250 620 -21000</td>
</tr>
</tbody>
</table>

| Expected Value: 4500 |

**d.** Discretize the distribution again, but this time into a two-branch node with probabilities of .5 and .5; then incorporate this node into the complete probability tree. Now, perform probability sensitivity analysis on the probabilities for platinum price by systematically changing the pair of probabilities—for instance, (1.0), (.9,.1), (.8,.2) and so on. At what probability of the high level of platinum price does your expected profit from holding the contracts become negative?

A two-branch discretization will come out with something like a .5 probability of $450 and a .5 probability of $600, for an expected value of $525.
Substituting this two-branch node into the tree for node 2 and then running probability sensitivity on node 2 produces the plot below. The Analyze Sensitivity Probability option of Supertree will help you create this plot. The bottom scale is the probability of the first branch of node 2, the $450 branch. Since the plot goes through zero expected value at a probability of low price of 1/3, the expected value is negative with probabilities of the high price greater than 2/3 (remembering that a contract to sell is a loser when the exercise price is less than the market price).

![Plot](image)

**e. Is this insight reflected in the influence diagram? Why or why not?**

The insight drawn from the sensitivity analysis occurs at the level of calculations. The influence diagram does not reflect this insight in its structure. The insight comes from varying the information contained in the influence diagram. Therefore, the influence diagram allows you to perform calculations and find the implication of each variation in information.

**2.15 Show that balancing areas in the discretization process is equivalent to choosing the expected value given that you are in the range. The expected value, given you are in the range $a \leq x \leq b$, is:**
\[ \int_{a}^{b} xf(x)dx \quad \frac{b}{a} \int f(x)dx \]

where \( f \) is the probability density, and the cumulative probability density is

\[ P(x) = \int_{-\infty}^{x} f(x')dx' \]

(Hint: you will probably need to do an integration by parts.)

We find from the definitions

\[ \frac{d}{dx} P_x(x) = \frac{d}{dx} \int_{-\infty}^{x} f(x')dx' = f(x) \]

Consequently

\[ \int_{a}^{b} f(x)dx = \int_{a}^{b} \frac{d}{dx} P_x(x)dx = P_x(b) - P_x(a) \]

and

\[ \int_{a}^{b} xf(x)dx = \int_{a}^{b} x \frac{d}{dx} P_x(x)dx \]

\[ = \int_{a}^{b} \left[ \frac{d}{dx} (x \frac{d}{dx} P_x(x)) - P_x(x) \right]dx \]

\[ = bP_x(b) - aP_x(a) - \int_{a}^{b} P_x(x)dx \]

Therefore, we find the expected value can be written

\[ \int_{a}^{b} xf(x)dx = \frac{bP_x(b) - aP_x(a)}{b} - \int_{a}^{b} P_x(x)dx \]

This result can be used, first, to prove the simplest case where \( a \) is less than the lowest value in the distribution and \( b \) is greater than the highest value, and, second, for the general case of any values of \( a \) and \( b \).
The graph below illustrates the simplest case.

The probability density \( f(x) \) is zero for \( x \not\in (a, b) \), which means that \( P(x < a) = 0 \) and \( P(x > b) = 1 \). For the expected value of the distribution, the results above give

\[
e = b - \int_a^e P_x(x)dx
\]

\[
e = b - \int_a^e P_x(x)dx - \int_e^b P_x(x)dx
\]

\[
e = b - \int_a^e P_x(x)dx + \int_e^b (1 - P_x(x))dx - (b - e)
\]

\[
\int_a^e P_x(x)dx = \int_e^b (1 - P_x(x))dx
\]

where the left side of the equation is the area \( A \) in the diagram and the right side is the area \( B \).
The general case is illustrated in the graph below.

We define four areas in this graph: $A, A', B,$ and $B'$, where $A$ and $B$ are (as before) the areas to be proven equal, and $A'$ and $B'$ are the respective areas between $A$ and $B$ and the top and bottom limits of the graph.

The expected value of the range is:

$$
e = \frac{bP_x(b) - aP_x(a) - \int_a^e P_x(x)dx + \int_e^b (1 - P_x(x))dx - (b - e)}{P_x(b) - P_x(a)}$$

$$\int_a^e P_x(x)dx = \int_a^b (1 - P_x(x))dx + P_x(b)[b - e] + P_x(a)[e - a] + e - b$$

The area $A'$ is described by $A' = P_x(a)(e - a)$, and $B'$ is described by $B' = (1 - P_x(b))(b - e)$. Adding and subtracting these areas from each side, we obtain the result we want.

$$\int_a^e P_x(x)dx - P_x(a)[e - a] = \int_a^b (1 - P_x(x))dx + P_x(b)[b - e - b + e] + P_x(a)[e - a - a + a] + e - b - b - e$$

$$\int_a^e P_x(x)dx - P_x(a)(e - a) = \int_a^b (1 - P_x(x))dx - (1 - P_x(b))(b - e)$$

where the left side of the equation is the area $A$, and the right side is the area $B$. 


3

Decisions Under Uncertainty

Problems and Discussion Topics

3.1  It is very common for people to judge the quality of a decision by looking at its outcomes. Is this an unbiased point of view? How can you best deal with uncertainty to make a decision?

Hindsight is biased—we know how any uncertainties were resolved. This makes it difficult to reconstruct the uncertainty at the time of the decision. It is easy to think that what actually happened was obvious at the time of the decision, but this is seldom the case. Focusing on the information available at the time of the decision and recording this information allows for a much fairer evaluation of the quality of a decision—including the situation in which potentially crucial information was ignored at the time of the decision. (“Clancy kept saying it probably would not hold, but we sent him out for doughnuts.”)

Students may have a variety of answers on how best to deal with uncertainty. These answers may range from explicit recognition and quantification of uncertainty to consulting Grandma’s corns. The authors use the former approach, including using decision analysis for personal financial decisions. We should note, however, that most personal decisions seem to require value trade-offs and clarification of risk attitudes rather than decision trees.

3.2  A good decision is defined as one that is logically consistent with the information, alternatives, and values brought to the decision. Give an example from your own experience (or perhaps from history) of a good decision/bad outcome and bad decision/good outcome. How could the bad decision have been improved? Remember that a good decision only has to be consistent with the information available at the time of the decision, though value of information should reveal the importance of the things you do not know.
ANSWERS TO PROBLEMS AND DISCUSSION NOTES

Possible examples from history are: good decision/bad outcome—sailing on the Titanic (Who would have thought that the safest ship afloat would sink on its maiden voyage?); bad decision/good outcome—assigning Michelangelo to paint the Sistine Chapel (Who would have thought that a sculptor could paint on such a vast scale?).

Many imaginative answers to this question are possible. To be correct, the answer need only properly identify the good or bad decisions or outcomes.

3.3 Describe the information, alternatives, values, and logic you used in deciding where to eat dinner the last time you went out. Afterwards, were you satisfied with the decision? Was the outcome good or bad?

Answers to this question should correctly separate the elements of a past decision into information, alternatives, values, and logic. For instance, information would include reviews and any personal or related experiences from having eaten at a particular restaurant. Alternatives should include the list of restaurants and, ideally, the “do nothing” alternative of eating at home. Values could include whether the student particularly likes fish, dislikes creamy sauces, does not want to do dishes tonight, or is or is not feeling fiscally adventurous. Logic could include such things as “Adrienne liked it, and I hate anything Adrienne likes,” or “It had a reasonable range of entree prices, given what I wanted to spend and the reviews.”

Satisfaction with the outcome depends on whether or not the student enjoyed the meal. However, satisfaction with the decision includes an assessment of whether or not the student was satisfied with the value tradeoffs of time, money, food preferences, and the like, and whether or not he or she felt that, given the information available at the time, another restaurant would have been a better choice.

3.4 How is getting the expected value different for decision trees than for probability trees?

The main difference is that, instead of simply taking the expected value at each (chance) node in a probability tree, in a decision tree a decision rule (either minimization or maximization) must be applied at each decision node to determine the rollback value for further calculation. Most trees maximize expected profit or minimize expected cost (or their certain equivalents) at each decision node.

3.5 Net present value (NPV) and expected value are abstract concepts in that people usually will not get the NPV of a cash flow (unless they are buying or selling an annuity) or the expected value of a lottery. NPV is usually understood as the result of a trade-off between future value and present value; expected value is regarded as a way to deal with or evaluate an uncertainty that is unresolved.

Write a short definition of each term. How are these concepts implemented in decision tree calculations?
Net present value is the value that results when positive and/or negative cash flows occurring at different times are discounted to produce present values, which are then summed. For any situation in which the particular discount rate used is appropriate, a person should be indifferent between receiving the actual cash flow over time or receiving its net present value. Net present value is implemented in decision trees by using an endpoint model, the final result of which is an NPV, which becomes an endpoint value at the end of the tree.

Expected value is obtained at a decision node by choosing the alternative with the maximum or minimum expected value and, at a chance node, by multiplying the different possible values by their corresponding probabilities and summing the resulting products. Expected value is implemented in decision trees by means of the rollback procedure in which expected values are calculated for the end nodes of the decision tree and then the results are used for further expected value calculations, etc., until an expected value has been calculated for the overall tree. With a risk-neutral attitude, a person should be indifferent between receiving the expected value of a tree or undertaking the uncertain venture represented by the tree.

3.6 List two radically different alternatives to getting your homework done (other than doing it yourself). Are these alternatives worth pursuing further? Why or why not? (Relate them to your values, the probability of getting caught, and the consequences of getting caught; to the effect when you take the midterm exam; etc.)

The question is aimed at producing creative alternatives (the hardest part of many decision problems) and then seriously evaluating them in light of tradeoffs among the relevant values. Students should recognize that, in any situation, there are many alternatives other than those first considered and that, depending on the values important in the decision problem, the “crazy” alternatives may have surprising advantages. Consider, for instance, the possibilities of simply not doing a homework assignment in your best subject and accepting the likely small loss of credit (if any) to allow more study time in a subject you are having trouble with—how might this satisfy a goal of maximizing overall grade point average?

3.7 Today, liability for damages is usually settled with a cash sum, whereas in the past, the rule was often blood for blood. (Consider Oedipus, for instance.) One rationale is not that the victim is being bought off (i.e., that money is equivalent to his or her pain), but rather that given that the incident has already occurred, the victim can use the money for some purpose whose value to him or her can help compensate for any losses. In addition, there are sometimes punitive damages to punish the malefactor.

Do you agree with this concept of trading money for pain and suffering? Do you find it more, less, or just as moral as the earlier method of compensation? Another alternative is no compensation. Suppose you are a victim. Do these two types of compensation mean anything different for you? Why or why not?
This question aims at provoking students to consider their own personal feelings about the implications and morality of trading intangibles for tangibles—a surprisingly frequent and thorny problem, even in business applications of decision analysis. Some people feel almost offended at the prospect of trading pain for money, for instance, while others find it a reasonable way to proceed in making business decisions. This may be a good representation, for instance, of what people do when choosing between buying aspirin or seeing a doctor—but other values such as fear, laziness, or distrust may also be operative. Instances when people accept money in advance to undergo pain are far more unusual.

One must remember that a prime requirement for a decision analyst is sensitivity and fidelity to the decision-maker’s values. Thus, within the bounds of logic, the decision-maker’s values and feelings must be respected and adhered to.

3.8 Anyone lending money at interest is establishing a marketplace between present and future money. The relationship between present and future is described by the interest rate. The marketplace works because some people have capital that would otherwise be unproductive if they did not lend it to people with a better use for it. List at least three reasons why people (or companies) might have different time values of money (discount rates).

Because this question is so conceptually thorny, instructors might consider requiring only one reason. Basically, applying a discount rate to obtain a net present value removes from consideration the element of timing of funds flows. Thus, for any present value, an infinite variety of cash flows can be generated to produce that present value, and the person or company whose discount rate was used should be indifferent among the present value and the cash flows.

For a person or company, those potential cash flows are opportunities to produce income in the future. The interest rate available through the financial markets gives the person or company a mechanism for converting those potential opportunities in a present-value cash sum.

If, in addition, there was a perfectly efficient market for capital (no barriers to entry or transactions, etc.), then everybody would be using the same discount rate (the market interest rate), and everybody would be indifferent between the present value and any of the potential cash flows.

However, problems can arise from a variety of sources. First, there may be effectively no market for capital for a company (such as one that has declared bankruptcy), or the market may be inefficient (such as when there are restrictions in trading a particular kind of bond). Second, there may be varying levels of transaction costs—e.g., a brokerage house can probably issue more of its own stock more cheaply than a non-brokerage house. Third, there may be differing beliefs about the future cash flows: other companies think that a particular market will grow slower and that sales will be lower than the company in question does.
The effect of these problems is seen in the ease/cost of borrowing or lending. A party can easily borrow when 1) it has opportunities that it is quite adept at converting into large future sums; 2) anybody can take advantage of the conversion opportunity, making the opportunity readily transferable; and 3) the parties share belief in the values to be realized in the future. Similarly, a party can (and generally will) lend only when the present sum obtainable by lending is higher than the sum available to it via any of its own conversion opportunities. The following examples explore these relationships:

- The federal government (through the IRS) is tremendously efficient at converting tax liability to dollars. The amount and consistency of this conversion make it easy for the government to borrow and lower the interest rate, even though the conversion opportunity is practically nontransferable (no one else can collect taxes).
- A gold trading company with gold reserves should find it easy to borrow against those reserves because they are easily converted by others, but the value of that conversion is affected by expectations for the future price of gold. A gold trading company may thus have a discount rate different from the interest rate.
- A diamond trading company should have a discount rate about equal to the available interest rate because its diamond assets are readily transferable and not subject to wide variations (thanks to de Beers), making it easy for the company to borrow against them.
- A movie company may have difficulty in borrowing and a high discount rate because its conversion opportunities consist of films to make, which have a highly uncertain return (hit or miss) and are not easy to transfer. (You have to be a movie production company to make use of them.)
- A real estate trading company with extensive holdings in booming areas should have an easy time borrowing if the appreciation has been steady.
- A pension fund may find it easy to borrow if its assets are readily tradable securities and bonds and its contributions are steady, or it may have difficulty if those assets are committed to payout obligations and protected from creditors (making them untransferable). The pension fund may be willing to lend at lower rates (lower discount rate) if it is restricted to conservative, passive investments, making its own conversion opportunities quite limited.
- Tax credits or liabilities may raise or lower a company’s discount rate, depending on whether or not it will be able to take advantage of the credit or ever have to pay the liability (the conversion factor) and whether these credits or liabilities can be
As with discount rates, people can have different attitudes to trading off certainty and uncertainty. List at least three reasons why people might have different attitudes toward risk. For instance, some people have dependents and some don’t. Why might a person’s risk attitude change over time? Can education play a role in this?

Here, the common thread is varying cash requirements (and their predictability) over time and, to some degree, varying asset levels. People react differently to decreases or increases in their asset levels. Decreasing assets push you closer to your perceived “subsistence” level, engendering increasingly strong aversion to risk, while greater assets have a diminishing effect, making it less likely you will hit subsistence levels. Thus, the following factors may have an effect:

- A person with mortgage payments may be less willing than a renter to tolerate uncertainty in cash flow because of the potential loss of the house if payments are not made.
- People may be willing to accept less uncertainty as they buy houses, have children, or get older. The possibility of shortfall could threaten one’s ability to maintain responsibilities. Similarly, as the years go by and the number of years left to work declines, one’s earning potential may decline, making it more difficult to overcome any possible shortfalls.
- Education can play a double role in changing a person’s attitude toward uncertainty. First, it can increase future earnings potential (decreasing uncertainty about being able to pay back loans) and making MBA students willing, for instance, to undertake large debt while in school in expectation of the higher future earnings. Second, education about uncertainty itself (such as in decision analysis) can help a person analyze uncertainty more rationally and thereby reduce aversion to it. Thus, a common phenomenon among decision analysis students is that they reduce the amount of insurance they purchase (or increase the deductible) as they learn to evaluate uncertainty and become more willing to accept it.
- Finally, changes in asset level may affect the willingness to undertake uncertainty. A person with $100,000 in the bank may be willing to take a new job with a risk of no pay for a year or two (and the chance of profitable stock options), while a new college graduate with $20,000 in student loans would typically be unwilling to do so.

List some of the principal values that must be considered in making decisions in a profit-making, publicly held hospital. Suggest the structure of a value model.
that establishes trade-offs between the values (at a deterministic level). State the limitations of the model. Are there ethical questions that cannot be simply resolved?

Some of the values competing for consideration include:

<table>
<thead>
<tr>
<th>Monetary Values</th>
<th>Nonmonetary Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profits</td>
<td>Doctor caseload</td>
</tr>
<tr>
<td>Stock price</td>
<td>Consult availability</td>
</tr>
<tr>
<td>Dividends to shareholders</td>
<td>Nurse patientload</td>
</tr>
<tr>
<td>Cost of care to patients</td>
<td>Comprehensiveness of care</td>
</tr>
<tr>
<td>Funds for equipment</td>
<td>Application of new techniques</td>
</tr>
<tr>
<td>Charges to doctors</td>
<td>Ability to attract doctors</td>
</tr>
<tr>
<td>Funds for residency programs</td>
<td>Continuing doctor and staff training</td>
</tr>
<tr>
<td>Percent insurance reimbursement</td>
<td>Reputation in community</td>
</tr>
<tr>
<td>Facility utilization</td>
<td>Ability to offer care for indigents</td>
</tr>
<tr>
<td></td>
<td>Timeliness of care</td>
</tr>
</tbody>
</table>

This list only suggests the complexity of the problem. Some of the items on the list—e.g., application of new techniques—can pose difficult ethical questions. Relating these factors in an explicit, deterministic value model would require the involvement of all constituencies, including: administrators, doctors, health care staff, facility staff, patients, and shareholders. Further, such a model would be intrinsically limited because each new patient would bring in his or her own value trade-offs among costs and health care features. However, one might venture a suggestion that the communication and discussion engendered by such an effort would contribute greatly to this ongoing and agonizing debate.

**3.11** Explain how (if at all) the purpose of an influence diagram is different from that of a decision tree. Would it ever make sense to draw a tree and then the influence diagram? If so, when?

Because the influence diagram and the decision tree are representations of the same problem, there is no essential difference between them. The influence diagram focuses more on the “big picture,” while the decision tree gives more emphasis to the details of how things link together. In a highly structured or very asymmetric situation, it may make sense to start out with the decision tree and then switch to an influence diagram as detail is added. Examples of this type of problem occur in research and development (R&D) and in litigation analysis.

**3.12** Can an equivalent decision tree always be drawn from an influence diagram and vice versa, or are there structural and/or informational differences that would make it impossible to do so without changing the problem or adding information? If there are differences, illustrate them with examples.
As formulated in this book, influence diagrams are equivalent to decision trees. The four rules for constructing influence diagrams ensure that they can be made, in technical mathematical terms, "decision tree networks."

However, there is one type of influence diagram that cannot be represented as a tree unless it is manipulated into a new form or until some side calculations are done. This is the case in which an arrow still points to the left-hand side of the page after you have followed the procedure for drawing a tree from an influence diagram. The influence diagram below illustrates this.

There is no way in the tree to show the probabilities for Survey Result because its probabilities depend on which branch of Outcome you are on, and Outcome occurs later in the tree.

One solution to this apparent problem is to manipulate the influence diagram. This process is discussed in the rules for manipulating influence diagrams in Chapter 11. For the example above, the manipulation reverses the arrow between Survey Result and Outcome. When this is done, the influence diagram becomes a "decision tree network," and the probabilities can be shown on the tree.

Instead of manipulating the influence diagram, you could also perform a side calculation using Bayes' Rule to obtain the probabilities to display on the tree. (Bayes' Rule is discussed in Chapter 4.) This calculation is completely equivalent to reversing the arrow in the influence diagram.

3.13 There are two main reasons for doing probabilistic sensitivity analysis in a decision-making process. First, probability assignments could change because of new information or could be different if there is more than one decision-
maker. In each case, we need to know how the decision will change given changes in probabilities (information).

Second, probability sensitivity can distinguish the major uncertainties that are most influential to the decision from those that are less influential. This may increase understanding of the decision, provide directions for further information-gathering activity, etc.

Interpret Figure 3–8, the probabilistic sensitivity plot. If there are crossover points between alternatives, how do you interpret the corresponding crossover probabilities?

In Figure 3–8, there are a number of crossover points between the alternatives. However, the only crucial ones are those that change the preferred decision. Thus, we are mostly interested in points at which the alternative with the highest expected value changes. Looking at the graph, we see that when the probability of the lowest level of cost ($200,000) is approximately .45 or greater, the $500,000 bid is preferred. When the probability of low cost is less than .45, the $700,000 bid is preferred. Accordingly, the .45 crossover probability is interpreted as follows: if you believe that there is at least a .45 chance of obtaining the low cost, you should bid $500,000; if you believe that there is less than a .45 chance of obtaining the low cost level, you should bid $700,000.

3.14 Draw an influence diagram that corresponds to the tree in Figure 3–10. (Hint: Introduce a deterministic node called Information Learned About Anode Bid.)

The influence diagram below shows the decision on whether to purchase perfect information on the Anode Bid. The arrow from the decision node to the profit node reflects the necessity of including the cost of obtaining the information in the Profit calculation.

The logic contained in the deterministic node is shown in the distribution tree below. Note that there are no rectangles or circles in this distribution tree because there are no alternatives or uncertain outcomes for the node—only logic.
3.15 Suppose you are going skiing this weekend for the first time. However, you are worried about the possibility of breaking your leg during your first time out. Your alternative is to go to the beach. After thinking about the possibilities, you have decided that you value a weekend of skiing with no mishaps at $1,000 and that you value a broken leg at $-5,000. Going to the beach instead is worth $500 to you. Finally, after talking to other people, you peg the chance of breaking your leg at 1 in 100.

a. Draw the influence diagram for this problem.

One way to draw the influence diagram is shown below. In this formulation, the probability of a broken leg depends on whether or not you go skiing.

A better way of drawing the diagram reflects the consideration that the probability of your breaking a leg while skiing is not dependent on your decision but on your skiing ability. The diagram below is a better representation of the problem as stated.

b. Structure the decision tree for this problem. What is the preferred decision and expected value?
As shown below, the preferred decision is to go skiing, with an expected value of $940.

<table>
<thead>
<tr>
<th>Destination</th>
<th>Exp Val</th>
<th>Probs BreakLeg</th>
<th>Exp Val</th>
</tr>
</thead>
<tbody>
<tr>
<td>Skiing</td>
<td>940</td>
<td>.010 Yes</td>
<td>-5000</td>
</tr>
<tr>
<td>Beach</td>
<td>500</td>
<td>.990 No</td>
<td>1000</td>
</tr>
</tbody>
</table>

Expected Value: 940

When the order of nodes has been reversed, as shown in the tree below, Supertree has had to symmetrize the tree. Note that whether or not you would have broken your leg skiing, the “beach” value is the same. The value of information is 995 – 940 = $55. The value of control is 1,000 – 940 = $60.

<table>
<thead>
<tr>
<th>Probs BreakLeg</th>
<th>Exp Val</th>
</tr>
</thead>
<tbody>
<tr>
<td>.010 Yes</td>
<td>500</td>
</tr>
<tr>
<td>.990 No</td>
<td>1000</td>
</tr>
</tbody>
</table>

Expected Value: 995

d. **What impact might the values of uncertainty and control have on your choice of weekend?**

The problem statement should have read “values of information and control”: Value of control—You might be willing to spend $50–60 for a skiing lesson your first time out if you thought that the lesson would completely eliminate the chance of a broken leg. Value of information—You might be willing to spend $50–60 for a good appraisal of your skiing abilities and for information on which trails would be safe and within your capabilities.

e. **For what probabilities of breaking your leg do you prefer going to the beach?**

As shown in the plot below, the expected value of going to the beach is constant as the probability of breaking a leg increases, while the expected value of going skiing drops off drastically. For probabilities of breaking a leg greater than about .1, going to the beach is preferred.
You are considering four different restaurants for dinner with a group of friends tonight. However, before deciding whether or not to eat at a restaurant, you’ll want to look at the menu and see what they have. At that point, you can either stay and eat or move on to the next restaurant. You value the cost of gathering everybody up and driving to the next restaurant at $20.

When you look at a menu, you have a scale in mind for rating it. You’ll assign a score of A, B, or C and value the scores for their contributions to the evening’s enjoyment as follows.

<table>
<thead>
<tr>
<th>Score</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$100</td>
</tr>
<tr>
<td>B</td>
<td>$50</td>
</tr>
<tr>
<td>C</td>
<td>$0</td>
</tr>
</tbody>
</table>

You judge your four possible choices as equally likely to get any of the three scores.

a. Draw the influence diagram and decision tree for this problem.

This is one of the structurally asymmetric situations in which the tree is much easier to draw and understand than the influence diagram.

In the schematic tree below, the decision nodes are understood to be replicated at the end of each of the tree branches of the preceding uncertainty node. There are 192 paths through the tree, and the compactness of the
schematic tree makes a drawing of reasonable size possible.

The influence diagram is in some ways less informative. The asymmetric structure of the problem is hidden in the logic of the Value node.

b. What is the best strategy for picking a restaurant, and what is the overall expected value?

This question could be answered using the brute-force method of evaluating the whole tree with four uncertainties and three decisions, as below. Note that the endpoint nodes are simple Basic-syntax expressions that set the value according to the restaurant rating.
A computationally easier but more sophisticated method would be to recognize that the expected value of any restaurant is $50, which, when considered with the $20 cost of moving, means that the optimal strategy is to stay with an A or B rating and to leave with a C rating. This strategy can be seen in the partial tree drawing below.

![Partial Tree Drawing](image)

A challenging assignment would be to find the expected value without using the complete tree.

c. **Suppose you can buy a restaurant guide for $20 that will alert you to any C restaurants. Modify your influence diagram and tree to reflect this choice. Should you buy it?**

This situation introduces a new decision before the first stay/go decision: Should you buy the restaurant guide? If you buy the guide, you will presumably pick the best of the four restaurants and go there directly; if there is a tie for best, you will use some criterion to break the tie.

The influence diagram is now very complex, as seen below. The deterministic node Restaurant Information Learned contains the logic that takes you directly to the desired restaurants in their order of desirability. The probabilities for the ratings are conditional on the information learned.
The decision tree appears much simpler, as seen below. The probabilities for the ratings for the alternative in which the guide is purchased will depend on the information from the guide.
There is a simpler way to evaluate the alternative in which the guide is purchased. If the guide said that all the restaurants were C restaurants, then the expected value of going to any one would be zero, and, of course, you would stay at whichever one you first went to (going to another would result in a value of \(-$20\)). The probability that this would occur is \(\frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{27}\).

If the guide said that any one or more of the restaurants was not a C restaurant, then the expected value of that restaurant would be \(.5(100) + .5(50) = $75\). The preferred strategy would then be to pick one of these restaurants and stay there since the expected value of the next restaurant would be \(75 - 20 = $55\). The probability of this occurring is \(1 - \frac{1}{27} = \frac{26}{27}\).

Thus, the value with the guide is \((\frac{1}{27})(0) + (\frac{26}{27})(75) = $7222\).

The value of the guide, then, is the expected value with the guide minus the expected value without the guide (\(72.22 - 64 = $8.22\)), which is less than the price of the guide. Thus, you should not buy it.

3.17 Raquel Ratchet is working on a Volkswagen Beetle to be sold at a collectors' car sale on Saturday. If she finishes it in time, she'll be able to sell it for $1,000, at a cost of $100 in parts. (She got the car from a junkyard.) She thinks there’s a 60 percent chance of being able to do this.

She also has the option of installing a turbocharger and intercooler in the car, which would quadruple the horsepower and enable her to sell it for $10,000. This would cost an additional $1,000 in parts. She thinks there’s only a 20 percent chance of being able to finish this amount of work by Saturday. If she misses the collectors’ sale on Saturday, Raquel figures she can only sell the car for $400 ($1,400 with turbocharger and intercooler).

Suddenly, the Wizard appears in the form of a good salesman and tells her that he can increase the selling price to $20,000 if she installs the turbocharger and intercooler. What is the maximum Raquel should be willing to pay the Wizard to do this? What is the maximum amount she should be willing to pay him on a contingency basis (if he makes the sale)?

a. Draw the influence diagram and tree for this problem.

The influence diagram is shown below.

![Influence Diagram](image-url)
The tree as entered in Supertree is shown below.

### CHAPTER 3 DECISIONS UNDER UNCERTAINTY

The tree as entered in Supertree is shown below.

```
<table>
<thead>
<tr>
<th>Turbo Rewards</th>
<th>Exp Val</th>
<th>Probs</th>
<th>Finish</th>
<th>Exp Val</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes</td>
<td>-1000</td>
<td>0.200</td>
<td>Yes</td>
<td>9900</td>
</tr>
<tr>
<td>No</td>
<td>0</td>
<td>0.800</td>
<td>No</td>
<td>1300</td>
</tr>
<tr>
<td>Yes</td>
<td>3020</td>
<td>0.200</td>
<td>Yes</td>
<td>900</td>
</tr>
<tr>
<td>No</td>
<td>420</td>
<td>0.800</td>
<td>No</td>
<td>300</td>
</tr>
</tbody>
</table>
```

b. **How much should Rachel pay in advance (nonrefundable) if there’s only a .8 probability of his making the sale?**

If the Wizard could increase the selling price with the turbocharger to $20,000, Raquel would have a .2 chance of achieving the $10,000 increase in selling price, so she should be willing to pay the Wizard a maximum of .2 \( \times \) 10,000 = $2,000.

On a contingency basis, Raquel should be willing to pay the Wizard at most $10,000 (the increase in selling price).

If there is only a .8 probability of making the sale, the most Raquel should pay the Wizard in advance is 3,276 – 2,020 = $1,256, as seen below. (The numbers in the ovals are the expected value of the portion of the tree to the right of that point.)

3.18 **Samuel Steelskull is thinking about whether or not to wear a helmet while commuting to work on his bicycle. He figures his chances of dying in an accident during the coming year without the helmet are about 1/3,000. The odds of dying go down to 1/5,000 with the helmet.**

Sam figures it is worth about $80 to him for his hair not to be messed up when he gets to work during the coming year, and he is pretty much indifferent between wearing and not wearing the helmet.

a. **Draw the influence diagram and tree for this problem.**

The influence diagram for this rather grim problem is quite simple, as seen below.
The tree is also quite simple. The one value that is not given in the problem is the value associated with death; this is shown an \(-L\) in the tree.

\[ \begin{align*}
\text{Wear Helmet} & \quad \text{Die} \\
\text{Yes} & \quad \text{Yes} -L/5000 \\
& \quad \text{No} -80 \\
\text{No} & \quad \text{Yes} -L/3000 \\
& \quad \text{No} 0
\end{align*} \]

b. What’s the implicit value Sam is putting on his life (i.e., a value that might be used in very small probability situations)?

This problem does not attempt to put a value on life. Most people in most circumstances would pay a very large sum to avoid certain death. On the other hand, most people would pay a relatively small sum to avoid a one-in-a-million chance of death. In this problem, there is a small probability of death. We use this situation to establish an implicit value that might be used in other problems where there is a small probability of death. (An interesting analysis of problems of this type can be found in “On Making Life and Death Decisions,” by R. A. Howard, Societal Risk Assessment, Plenum Publishing Corporation, 1980, pp. 89-113, reprinted in Ronald A. Howard and James E. Matheson, eds., Readings on the Principles and Applications of Decision Analysis, Strategic Decisions Group, Menlo Park, CA, 1984, pp.483-500.)

The tree above shows Sam’s decision. Since he is indifferent between wearing and not wearing a helmet,

\[ \begin{align*}
(-L/5000) - 80 &= (-L/3000) \\
(L/3000) - (L/5000) &= 80 \\
(5L - 3L)/15000 &= 80 \\
2L &= 1,200,000
\end{align*} \]
3.19 Your company has recently developed Chewsy, a new sugar-free gum that contains fluoride. Not only does it taste good, it's also good for your teeth. You are faced with the decision of whether or not to introduce Chewsy to the market.

The total sales of chewing gum are expected to be about $200 million over the next 10 years.

Your marketing personnel feel that with their best efforts and with a front-end marketing expenditure of $4 million, your company could capture from 2 percent to 10 percent of the chewing gum market with Chewsy. They have given you the following probabilities.

<table>
<thead>
<tr>
<th>Market Share</th>
<th>Probabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>High (10%)</td>
<td>.30</td>
</tr>
<tr>
<td>Medium (6%)</td>
<td>.50</td>
</tr>
<tr>
<td>Low (2%)</td>
<td>.20</td>
</tr>
</tbody>
</table>

Your financial advisors point out that the profit margin on Chewsy is quite uncertain because of unusual manufacturing requirements. They say that there is a 40 percent chance that the profit margin will be only 25 percent of sales revenue and a 60 percent chance it will be 50 percent of sales revenue. The manufacturing requirements will be known before the marketing decision needs to be made.

a. Draw the influence diagram for this problem.

b. Structure the decision tree for Chewsy. What is the preferred decision and expected value?

The structure for this tree is shown below. Following it is a drawing of the tree showing the overall expected value of $1.12 million from choosing to market the gum.
c. Plot the probability distribution for profit for the Chewsy decision. What are the expected values for both alternatives?

As noted above, the expected value for the market alternative is $1.12 million, while the expected value for doing nothing is, of course, zero.

d. What is the value of information on market share? On profit margin? On both?
The value of information on market share is $1.6 - 1.2 = $0.4 million, or $400,000.

The value of information on profit margin is $1.44 - 1.2 = $0.24 million, or $240,000.

The value of information on both is $1.8 - 1.2 = $0.6 million, or $600,000. Note that the value of information on more than one uncertainty is not necessarily the sum of the individual values of information. The sum of the value of information for each uncertainty is $400,000 + $240,000 = $640,000, but the value of information on both together is $600,000, which is $40,000 less.

e. What is the value of control on market share? On profit margin? On both? Are there any possible ways of achieving further control over either of these uncertainties?

As you can see from the value of information tree for market share, the best possible outcome is $4 million, making the value of control on market share $4 - 1.2 = $2.8 million. The value of information tree for margin shows that the best possible outcome is $2.4 million, making the value of control on margin $2.4 - 1.2 = $1.2 million. From the tree for information on both, you find that the best outcome is $6 million, making the value of controlling both $6 - 1.2
$4.8 million.

Some control might be achieved over market share, for example, by advertising expenditures, while control could be achieved over margin by constructing a pilot manufacturing process to determine the manufacturing cost and then exploring ways to control the highest cost components (invest in more automated equipment, produce in a country with lower labor costs, etc.).

\[ t. \quad \text{Does the preferred decision vary with the probabilities for market share or margin? What decisions are preferred for what ranges of probability?} \]

The plot below shows the sensitivity to probabilities for market share. When the probability of the high market share (10 percent) is around .4 or greater, the market alternative is preferred. For probabilities below that [probability of the low (2 percent) market share greater than .6], the alternative not to market is preferred.

The sensitivity to probabilities for margin is shown below. When the probability of the low margin (25 percent) is greater than around .75, the alternative not to market is preferred. When the probability is less than this [probability of the higher (75 percent) margin is greater than .25], the market alternative is preferred.
3.20 The Southern Power Company is planning to submit a major rate increase request to the state Public Utilities Commission. The Commission has assured Southern Power that it would approve a request for a moderate rate increase. With this moderate rate increase, Southern would receive $40 million in additional revenues during the next few years (relative to no rate increase).

However, the company is also considering a riskier course of action—requesting a high rate increase that would yield $100 million in additional revenues, if approved. If the high rate increase is not approved, there is still some chance the Commission would grant Southern a low rate increase, which would mean $30 million in additional revenues. Of course, the possibility exists that the Commission would simply refuse any rate increase whatsoever if Southern asks for the high increase.

The best information within the company indicates a 70 percent probability the Commission would disapprove the high rate increase request. Given that it does so, the chance it would then grant a low rate increase is believed to be 60 percent.
ANSWERS TO PROBLEMS AND DISCUSSION NOTES

a. Draw the influence diagram for Southern's problem.

![Influence Diagram]

b. Draw the decision tree for Southern's decision problem.

![Decision Tree]

Expected Value: 42.60

<table>
<thead>
<tr>
<th>Request</th>
<th>High</th>
<th>Moderate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exp Val</td>
<td>42.60</td>
<td>40.00</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Prob</th>
<th>Approval</th>
<th>Exp Val</th>
</tr>
</thead>
<tbody>
<tr>
<td>.300</td>
<td>Yes</td>
<td>100.00</td>
</tr>
<tr>
<td>.700</td>
<td>No</td>
<td>18.00</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Probs Second Approval</th>
<th>Exp Val</th>
</tr>
</thead>
<tbody>
<tr>
<td>600</td>
<td>Granted</td>
</tr>
<tr>
<td>400</td>
<td>Denied</td>
</tr>
</tbody>
</table>

c. Find the expected value of each alternative.

See the tree in 3.20b.

d. Draw the probability distribution on profit for each alternative.

![Probability Distribution]
e. Calculate the value of perfect information on:

- Whether or not the Commission approves the high rate increase

The value of information on the approval of the high rate increase is 58 – 42.60 = $15.4 million.

<table>
<thead>
<tr>
<th>Probs Approval Exp Val</th>
<th>Request Exp Val</th>
</tr>
</thead>
<tbody>
<tr>
<td>High 100.00</td>
<td>Moderate 40.00</td>
</tr>
<tr>
<td>No 40.00</td>
<td>Moderate 40.00</td>
</tr>
</tbody>
</table>

- Whether or not the Commission would grant a low rate increase given that it does not approve the high rate increase

The value of information on approval of a low rate increase if the high request is denied is 46.60 – 42.60 = $4 million.

<table>
<thead>
<tr>
<th>Probs Second Approval Exp Val</th>
<th>Request Exp Val</th>
</tr>
</thead>
<tbody>
<tr>
<td>High 51.00</td>
<td>Moderate 40.00</td>
</tr>
<tr>
<td>No 40.00</td>
<td>Moderate 40.00</td>
</tr>
</tbody>
</table>

Note that Supertree’s symmetrization of the tree (because the order of nodes has been changed) may make this partial tree drawing appear confusing, but the values have been properly symmetrized.

- Both of the above.

The value of information on both approvals is 58 – 42.6 = $15.4 million.

3.21 Your company markets an all-purpose household glue called Easystick. Currently, a sister company in another country supplies the product at a guaranteed delivered cost of $2.00 per unit. You are now thinking about producing Easystick locally rather than continuing to import it. A staff study indicates that with a projected sales volume of 4 million units over the product’s life, local production would cost an average of $1.50 per unit.

However, two things could significantly affect this cost. First, the government in your country is considering imposing a heavy tax on the primary raw material of Easystick. This would increase the average production cost of
Easystick to $2.25 per unit. You think there is a 50/50 chance the government will impose the tax.

The second factor is a newly developed improvement in the production process that uses the expensive raw material more efficiently. This new process would reduce the average production cost of Easystick as shown below.

<table>
<thead>
<tr>
<th>Average Cost Per Unit</th>
<th>Old</th>
<th>New</th>
</tr>
</thead>
<tbody>
<tr>
<td>Process</td>
<td>$1.50</td>
<td>$1.25</td>
</tr>
<tr>
<td></td>
<td>$2.25</td>
<td>$1.75</td>
</tr>
</tbody>
</table>

Unfortunately, local conditions may make it impossible to implement the new process. Your staff estimates a 60 percent chance of being able to use the new process.

Should you continue to import or switch to local production?

a. Draw the influence diagram for this problem.

b. Draw the decision tree for this problem and calculate the expected value for each option. (For the outcome measure, use the total savings in cost relative to importing Easystick.)
c. Draw the probability distribution on profit for each option.

![Probability Distribution Graph](image)

\[ \text{Cumulative Probability} \]

\[
\begin{array}{cccc}
\text{Savings} & -2.00 & -1.00 & 0.00 & 1.00 & 2.00 & 3.00 & 4.00 \\
\text{Produce; } EV = 1.40 & & & & & & & \\
\text{Import; } EV = 0.00 & & & & & & &
\end{array}
\]

d. Calculate the values of perfect information and control on:

- Whether or not the tax will be imposed on the raw material

The value of information on whether the tax is imposed is \( 1.40 - 1.40 = 0 \). From the tree drawing, you can see that the decision is to produce regardless of whether or not the tax is imposed. We expect a value of information of zero when the best alternative is the same regardless of the information.

<table>
<thead>
<tr>
<th>Probs</th>
<th>Tax Exp Val</th>
<th>Material Exp Val</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes</td>
<td>0.20</td>
<td>Produce 0.20</td>
</tr>
<tr>
<td>No</td>
<td>2.60</td>
<td>Import 0.00</td>
</tr>
</tbody>
</table>

Expected Value: 1.40

- Whether or not the new production process can be used

Here, too, the preferred decision is the same in either case, and the value of information is zero.

<table>
<thead>
<tr>
<th>Probs</th>
<th>Process Exp Val</th>
<th>Material Exp Val</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes</td>
<td>0.50</td>
<td>Produce 0.50</td>
</tr>
<tr>
<td>No</td>
<td>2.00</td>
<td>Import 0.00</td>
</tr>
</tbody>
</table>

Expected Value: 1.40
• Both of the above.

Only when you know the outcome of both uncertainties will you know in advance when the –1 outcome will occur and when importing will therefore be preferred. Accordingly, there is a value of information on both uncertainties at the same time of $1.6 - 1.4 = $0.2 million, or $200,000.

e. Do a probability sensitivity analysis to determine the preferred decision for different ranges of probabilities.

In the sensitivity to tax probability shown below, you can see that producing is preferred to importing for all probabilities of imposition of the new tax—a result that would be expected from the zero value of information on that uncertainty.

![Probability Sensitivity Chart]

Because there is zero value of information on production process (the preferred decision never switches), the sensitivity to the probabilities for
production process shown below shows no crossover point for the alternatives.

We know that there is value of information on both uncertainties: there is one scenario in which the preferred decision switches. An interesting exercise for students would be to plot the probability for the new tax on one axis and the probability for the new process on the other and show the regions of the plot in which producing or importing is preferred. Examples of this type of sensitivity can be found on pages XX and XX of the Decision Analysis for the Professional textbook.

3.22 It is 1986 and Shipbuilder, Inc., has decided to take a long, hard look at its telephone needs. Its present system has one major problem: it is very costly to make moves or changes. A task force has identified the most attractive alternative: Fone-Equip can install a system that will enable moves and changes to be made at almost no cost.

The five-year lease on the current system is up for renewal for the period 1987–1991. Another renewal of the lease would be made in 1991 for the period 1992–1996. Fone-Equip’s system is available only for outright purchase. Under any alternative, Shipbuilder will be in the same position in 1996 (a completely new phone system will be needed), so costs beyond 1996 can be neglected.

Shipbuilder is planning a shipbuilding program called Program A, which will entail considerable phone moves and changes in the years 1987–1991. Under the present system, these changes would cost $2 million per year. However, if Program A does not materialize, there will be very little in the way of phone moves and changes in this period.

Similarly, Program B would entail $4 million per year in costs for phone moves and changes in the years 1992–1996. However, if Program B does not materialize, there will be very few moves and changes in this period.

All costs were estimated in 1986 dollars, with the effects of inflation removed. The costs for the present system are $1.5 million per year lease and $0.5 million per year recurring costs. Fone-Equip’s system has a purchase price of $14 million (including installation) and $1.0 million per year recurring cost.

Shipbuilder judged that Program A has a 50 percent chance of occurring. The 1986 telephone decision will be made before Program A’s fate is known.

Program B is judged to have a 60 percent chance of occurring if Program A does occur, but only a 30 percent chance of occurring if Program A does not occur. If there is a telephone decision to be made in 1991, it will be made before Program B’s fate is known.

There is also a 20 percent uncertainty on what Fone-Equip’s system would cost in 1991.

The decision-maker insisted on using a discount rate of 10 percent. At this rate, the net present value in 1986 is:
• $1 expense in 1987—$0.91
• $1 expense in 1992—$0.56
• $1 expense per year for 1987–1991—$3.79

a. **Draw the influence diagram and decision tree for this problem.**

In the influence diagram and tree below, the Fone-Equip system cost uncertainty in 1991 is modeled as an uncertain multiplier on the projected cost.

```plaintext
Program A -> Decision, 1986
/                  \                  
/                     \                     
Program B -> Decision, 1991
| Fone-Equip System Cost Multiplier, 1991 |
|                                  NPV |

Decision, 1986 Program A
| Yes | No |
--------|----|----|
Yes | .8 |
No | 1.2 |
```

b. **What is Shipbuilder’s best strategy? Show the distribution on costs for the alternatives.**

Although this problem can be solved entirely on paper, it has enough calculations to make it a good one for introducing the use of spreadsheets. The Excel spreadsheet shown below is a simple representation of this problem.
### Shipbuilder Decision Table

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
<th>Name</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Purchase phone system in 1986</td>
<td>yes/no</td>
<td>F_86</td>
<td></td>
</tr>
<tr>
<td>Purchase phone system in 1991</td>
<td>yes/no</td>
<td>F_91</td>
<td></td>
</tr>
<tr>
<td>Cost of phone system</td>
<td>14 $million</td>
<td>F_purch</td>
<td></td>
</tr>
<tr>
<td>Annual cost of phone system</td>
<td>1 $million/yr</td>
<td>F_ann</td>
<td></td>
</tr>
<tr>
<td>Cost multiplier, 1991 system</td>
<td>1.2</td>
<td>F_91_m</td>
<td></td>
</tr>
<tr>
<td>Annual Lease Cost</td>
<td>2 $million/yr</td>
<td>Lease</td>
<td></td>
</tr>
<tr>
<td>Program A</td>
<td>yes/no</td>
<td>P_A</td>
<td></td>
</tr>
<tr>
<td>Program A Cost</td>
<td>2 $million</td>
<td>P_A_Cost</td>
<td></td>
</tr>
<tr>
<td>Program B</td>
<td>yes/no</td>
<td>P_B</td>
<td></td>
</tr>
<tr>
<td>Program B Cost</td>
<td>4 $million</td>
<td>P_B_Cost</td>
<td></td>
</tr>
<tr>
<td>Discount Rate</td>
<td>10% disc rate</td>
<td>disc_rate</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Phone System Purchase Cost</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Phone System Annual Cost</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Phone Annual Lease Cost</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Program A Cost</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>Program B Cost</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Total Cost</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>17.8</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>NPV</td>
<td>-27.76</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The box at the top of the sheet contains all the input for the problem: column A contains the description, column B contains the value used in the calculations, column C contains the range name used for the values in column B, and column D documents the units used for the values in column B.

The calculations section in lines 20–29 use the values to calculate the cost. The calculations use formulas like =IF(F_86="yes",F_purch,0) to plug in a cost of $14 million if the 1986 Fone-Equip decision is yes and 0 otherwise. F_86 is a range name in the spreadsheet for the cell containing the yes/no key (with the name created from the label next to it) and a node name in SuperTree.

The following formulae define the time series in the Calculations section:

**Cells**

- B22 =IF(F_86="yes",F_purch,0)
- G22 =IF(AND(F_86="no",F_91="yes"),F_purch*F_91_m,0)
- B23:F23 =IF(F_86="yes",F_ann,0)
- G23:K23 =IF(OR(F_86="yes",F_91="yes"),F_ann,0)
- B24:F24 =IF(F_86="no",Lease,0)
- G24:K24 =IF(AND(F_86="no",F_91="no"),Lease,0)
- B25:F25 =IF(AND(F_86="no",P_A="yes"),P_A_Cost,0)
- G25:K25 =0
The resulting tree is shown below. As it should, it has identical values for the top part of the tree where the initial purchase decision is made (the uncertainties on projects A and B are then also irrelevant).
As can be seen from the tree, buying the Fone-Equip system in 1986 just edges out not buying the system in 1986, with an expected cost of $17.2 million versus an expected cost of $18.3 million for not buying. Also note that, depending on the scenario, the decision may be to either purchase or not purchase the system in 1991. Probability distributions for the two alternatives are shown below.
c. **What is the value of delaying the 1986 decision until Project A’s fate is known?**

This value is obtained by moving the Program A node of the tree in front of the Fone86 node and then calculating the difference between it and the original expected value: \((-15.4) - (-17.2) = 1.7\), for an expected cost saving of \$1.7\ million.

\[
\begin{array}{c|c|c|c}
\text{Probs} & \text{P}_A & \text{Exp Val} & \text{F}_86 & \text{Exp Val} \\
\hline
\text{Yes} & -17.2 & \text{Yes} & -17.2 \\
\text{No} & -22.8 & \text{No} & -13.7 \\
\end{array}
\]

\text{Expected Value: -15.4}

---

d. **What is the value of delaying the 1991 decision until Project B’s fate is known?**

Only the first node of the tree is shown below, but the Project B node was moved in front of the Fone91 node. The value of delaying the 1991 decision, then, is \((-16.9) - (-17.2) = 0.3\), for an expect cost saving of \$300,000.

\[
\begin{array}{c|c|c|c}
\text{F}_86 & \text{Exp Val} \\
\hline
\text{Yes} & -17.2 \\
\text{No} & -16.9 \\
\end{array}
\]

\text{Expected Value: -16.9}
e. Suppose Shipbuilder could obtain perfect information on Project A and Project B before the 1986 decision is made. What would this information be worth?

The value of perfect information on both projects is \((-14.9) - (-17.2) = 2.3\), or $2.3 million.

\[
\begin{array}{c|c|c|c|c}
\text{Probs} & \text{P A Exp Val} & \text{Probs} & \text{P B Exp Val} & \text{F_86 Exp Val} \\
\hline
\text{0.600} & \text{Yes} & -17.2 & \text{Yes} & -17.2 \\
\text{0.400} & \text{No} & -17.2 & \text{Yes} & -17.2 \\
\text{0.500} & \text{No} & -12.7 & \text{Yes} & -17.2 \\
\text{0.700} & \text{Yes} & -11.2 & \text{No} & -11.2 \\
\end{array}
\]

\[\text{Expected Value: -14.9}\]

f. Show how the expected values associated with the 1986 decision vary with the probability of Project A’s occurring.

As shown in the plot below, when the probability that Project A will happen is less than about .35, not purchasing the Fone-Equip system in 1986 is preferred (having a lower expected cost). When the probability that Project A will occur is greater than .35, purchasing in 1986 is preferred.
The Pharmaceutical Division of Dreamland Products has been the world leader in the area of soporific drugs. Its major product, Dozealot, is approaching the end of its patent life, and already sales have fallen significantly from the peak because of the inroads of new and superior competitive products. However, Dozealot sales are still quite significant and are considered to be of strategic importance for maintaining the sales of the entire product line of soporific drugs. Therefore, the research and development (R&D) department has defined two alternative approaches to improve the product quality and, thus, future sales prospects.

One approach, which is quite conventional, is simply to reformulate the product to minimize an undesirable side effect that exists in the current galenical form. The manager in charge of galenical development has developed a number of new formulations since the original introduction of Dozealot that have the desired characteristic, but even such a simple change in the formulation will require a development expenditure of 500,000 Swiss francs. By estimating the increase in sales of both Dozealot and the rest of the soporific product range, the product manager for soporific drugs has estimated the value of this improvement. Taking into account the production cost of the new formulation and the minor investments required, the improvement in Dreamland’s cash flow would be substantial and yield a net present value (NPV) of 2.5 million Swiss francs (not including the cost of development).

The second approach, which is riskier but potentially more rewarding, involves a new controlled-release technology based on differential microencapsulation. This approach, if successful, would not only eliminate the undesired side effects but would also substantially improve the product efficacy. Market forecasts and cash-flow analyses indicate that this product, B, would be four times as profitable to produce and market as the more conventional product A, described above. The drawbacks, however, are that the microencapsulation development project would cost 3 million Swiss francs and still might fail because of an inability to control the differential layering process within tolerances specified by Good Manufacturing Practice. After a recent review of the microencapsulation process development efforts, the R&D director concluded that there is 1 chance in 2 of being technically successful within the deadlines imposed by the patent life of Dozealot.

a. Draw the influence diagram and decision tree for Dreamland Products considering it could separately pursue the development of A or B or even pursue both to reduce the risks involved in B. In the latter case, it would naturally market B and not A if B were a technical success.

The influence diagram for this problem is shown below. Note that the probability of success for Project B is modeled as independent of the decision of whether to pursue Project B.
b. **Compute the expected value of each alternative, assuming B has 1 chance in 2 of being successfully developed. Based on the criterion of expected value, what should Dreamland do?**

The tree below shows that developing both A and B is preferred, with an expected value of 2.75 million Swiss francs.

\[
\begin{array}{c|c|c}
\text{Develop Exp Val} & \text{Probs} & \text{ProjectB Exp Val} \\
\hline
A & .500 & \text{Success} \\
 & .500 & \text{Failure} \\
B & .500 & \text{Success} \\
 & .500 & \text{Failure} \\
A \& B & .500 & \text{Success} \\
 & .500 & \text{Failure} \\
\end{array}
\]

\[
\text{Expected Value: 2.75}
\]

Since the probability of successfully developing product B is difficult to determine, Dreamland’s managing director would like to know how sensitive the best decision is to this probability assignment.

c. **Compute the expected NPV of developing B alone and of developing A and B simultaneously as a function of } P_B, the probability of technical success of B.**

This step is not necessary if Supertree is used to perform the probability sensitivity analysis. However, the problem is simple enough that showing the equations is instructive.

- Expected value of A alone = 2
- Expected value of B alone = 10p_B – 3
- Expected value of A and B = 7.5p_B – 1

d. **Graphically represent the expected value of the three alternatives as a function of } P_B and determine for what range of values of } P_B each alternative is best.**
Resolving the technical uncertainties surrounding the microencapsulation project early could be achieved by immediately conducting a few critical experiments at an additional cost of 1 million Swiss francs.

\[ e. \quad \text{Compute the expected value of perfect information on whether microencapsulation could be successfully accomplished, assuming an initial probability of success of } 0.5. \]

The value of perfect information is \(4.5 - 2.75 = 1.75\) million Swiss francs, as can be seen in the tree below.

\[ f. \quad \text{Compute and graphically represent the expected value of the entire project given perfect information about whether or not microencapsulation would be feasible as a function of the initial probability of success } \frac{1}{2}. \text{ Graphically show the expected value of perfect information and determine over what range of initial probability of success } \frac{1}{2} \text{ this value exceeds 1 million Swiss francs.} \]

The expected value with perfect information as a function of \( \frac{1}{2} \) can be obtained by having Supertree do a probability sensitivity on the success of \( \frac{1}{2} \) with
Project B’s success node brought to the front of the tree. The resulting plot is shown below. Note that the requested scale is the same as in the plot above.

With the same scale on the plots, the line from the second plot can be superimposed on the first plot; this is the top line in the graph below. The vertical distance between this line and the line showing the value of the best alternative is the value of perfect information as a function of $P_B$. 
3.24 The internal rate of return (IRR) of a venture is the value that, if substituted for the discount rate, makes the net present value (NPV) zero. There are several difficulties in using IRR as the sole criterion in choosing between alternatives.

Consider the following two ventures:

A: Invest I in year 0, receive positive cash flow, C, in all years from year 1 to infinity

B: Invest I in year 0, receive positive cash flow, K, in year 1, nothing in succeeding years.

a. Solve for \( c = C/I \) and \( k = K/I \) in terms of \( n = NPV/I \) and \( d \), the discount rate. Use the sum

\[
\sum_{i=1}^{\infty} \frac{1}{(1+d)^i} = \frac{1}{d}
\]

For venture A, we have:

\[
NPV_A = -I + \sum_{i=1}^{\infty} \frac{C}{(1+d)^i}
\]

\[
NPV_A = -I + \frac{C}{d}
\]
\[ n_A = -1 + \frac{c}{d} \]
\[ c = d(n_A + 1) \]

For Venture B, we have:

\[ NPV_B = -1 + \frac{K}{(1 + d)} \]
\[ n_B = -1 + \frac{k}{(1 + d)} \]
\[ k = (1 + d)(n_B + 1) \]

b. Assume that both ventures have the same investment and the same NPV. Plot the IRR value against \( n \) for the two ventures. (Take the discount rate \( d \) to be .1.)

The IRR is the value of the discount rate for which the NPV is 0. For this problem, we assume \( n_A = n_B = n \). For venture A, we can find the IRR value \( d^* \):

\[ 0 = -1 + \frac{c}{d_A} \]
\[ d_A^* = c = d(n + 1) \]

For venture B, we find:

\[ 0 = -1 + \frac{k}{(1 + d_B)} \]
\[ d_B^* = (k - 1) = (1 + d)(n + 1) - 1 \]

The graph below shows the IRR value \( d^* \) for various values of NPV \( n \) for both ventures.
c. For equal n (equal value to the decision-maker), how do the IRR values of long-term and short-term investments (A and B) differ? Is IRR an adequate decision criterion? What assumptions do you have to make concerning use of funds after year 1 for investment B?

Take an attractive pair of ventures with, say, n_A = n_B = n = .5. These are two attractive ventures, both of which have the same value (NPV) to the decision-maker. Yet their IRR values, d*, are very different. IRR in this case does not, by itself, appear to be an adequate decision criterion.

Compare these ventures with ventures with the minimum acceptable value, n = 0. For long-term venture A, the IRR value for n = .5 is not very different from the IRR value for n = 0. For short-term venture B, the IRR values for n = .5 and n = 0 are very different. Even when comparing the same type (long or short term) of venture with different NPVs, IRR must be used with care as a decision criterion, since it behaves differently in the two cases.

In using either decision criterion (NPV or IRR), you have to make some assumption concerning the use of funds for short-term investment B in the years 2 through infinity. The implicit assumption in using NPV is that the funds can be reinvested at the discount rate d, thus yielding an NPV of 0 for these years; this assumption is not unreasonable because d represents the decision-maker’s time value of money and is presumably closely related to his or her cost of funds and average return on opportunities. The implicit assumption in using IRR is that the funds are not invested during these years and yield no return; this assumption is clearly inadequate.

3.25 A venture with an NPV of zero is acceptable to the decision-maker. Why is this true? What problems in interpretation does this cause for the decision-maker unfamiliar with the concept? What steps would alleviate these problems?

An NPV of zero meets the decision-maker’s time value of money. The investment returns what the decision-maker needs and desires. A positive NPV exceeds these needs and desires. However, at first blush, a zero value for the NPV appears to indicate no return for the investment; be sure that the decision-maker appreciates the fact that the investment is included in the calculation! Second, the zero NPV appears to be a judgment of “no value.” However, if the discount rate is well chosen, the NPV of all the company’s ventures should average out to around zero. Usually a reminder (or several reminders) of these facts is sufficient to avoid most problems.

3.26 If financing is available at an interest rate equal to the decision-maker’s discount rate, a venture with a positive NPV can be transformed into a venture with an infinite IRR.

a. How can this be done?

For simplicity, let us assume that the cash flow, C_t, in all years is positive and that the investment, I, occurs in year 0. This will make the point, and the
argument can be extended to cover more general cases. The NPV is defined to be

\[ NPV = -I + \sum_{i=1}^{\infty} \frac{C_i}{(1+d)^i} \]

If we can find a friendly banker with an interest rate equal to our discount rate, we can arrange for a loan for amount \( I \) with a time pattern the same as that for \( C_i \). The payments are \( aC_i \).

\[ I = \sum_{i=1}^{\infty} \frac{aC_i}{(1+d)^i} \]

We have now eliminated the negative cash flow of \( I \) in year 0 and replaced the cash flow in years 1 through infinity by \( (1-a)C_i \).

\[ NPV = \sum_{i=1}^{\infty} \frac{(1-a)C_i}{(1+d)^i} > 0 \]

Because the NPV is positive, we find that \( a < 1 \).

\[ \sum_{i=1}^{\infty} \frac{C_i}{(1+d)^i} > a \sum_{i=1}^{\infty} \frac{C_i}{(1+d)^i} \]

\[ a < 1 \]

This shows that the new cash flow, \( (1-a)C_i \), is positive in all years. This means the IRR is infinite—for zero investment you get a non-zero return! To see this, remember that the IRR is the value of \( d^* \) for which

\[ 0 = \sum_{i=1}^{\infty} \frac{(1-a)C_i}{(1+d^*)^i} \]

Because all the numerators in the sum are positive, the sum can be zero only if all the denominators are infinite. This means \( d^* \) is infinite.

b. Does a similar situation exist for leasing alternatives?

In leasing alternatives, one avoids the initial investment and pays an annual fee for the use of the product. Although there may be years of negative cash flow, the IRR of such alternatives can be very high or even infinite, and this is mostly an artifact of the method of financing chosen. The NPV, of course, reflects the true value of the alternative.

c. For existing businesses, one alternative usually involves no major new investments. What is the IRR for this strategy? Compare the type of results you might expect for NPV and for IRR for “Invest” and “No New Investment” strategies. In what situations might you prefer NPV or IRR as a decision criterion?
Alternatives with no major new investments typically have very high IRR values; if the cash flow in year 0 covers the investment, the IRR may be infinite. "Invest" strategies will have much lower IRR values since they have investments in year 0. The NPV for "No New Investment" alternatives has no large negative investment, but the cash flows are typically dwindling and the NPV may not be large. On the other hand, the NPV of "Invest" strategies has to balance the investment against the cash flow and may not be large either.

If you use IRR as a sole decision criterion, you will usually be steered in the direction of "No New Investment" strategies because of their very high IRR. NPV as a criterion will more truly reflect the value of the alternatives to the decision-maker, and you may find the IRR criterion misleading.

It is the authors’ experience that NPV is always adequate as a decision criterion. IRR, on the other hand, can frequently lead to problems. When the decision-maker requires the use of IRR, it is wise to report results using both criteria.
4

Probabilistic Dependence

Problems and Discussion Topics

4.1 What are joint probabilities? How are these different from conditional probabilities?

A joint probability describes the chance that two or more different events occur, while a conditional probability describes the chance that an event will occur given that one or more other events have already occurred. Consider the question “Will Joe Montana win for the 49ers on Sunday?” A joint probability would be $p(\text{Joe Plays and 49ers Win} \mid S)$, the probability that Joe plays and the 49ers win. A conditional probability would be $p(\text{49ers Win} \mid \text{Joe Plays and } S)$, the probability that the 49ers win given that Joe plays. (See Chapter 10 for definitions and descriptions of this notation.) Note that the unconditional and conditional probabilities are usually less than one (and never greater), and multiplication makes the joint probability even smaller.

$$p(\text{Joe Plays and 49ers Win} \mid S) = p(\text{49ers Win} \mid \text{Joe Plays and } S) \cdot p(\text{Joe Plays} \mid S)$$

The joint probability on the left is smaller than either of the two probabilities on the right. Another example of this effect: the chance that rain and ants will spoil a picnic is smaller than the chance of that either one alone will.

4.2 Suppose your interview with a decision-maker has revealed a dependency between two uncertainties. How do you determine which uncertainty depends on the other for assessment purposes? What does this imply for the order of assessment?

For the purposes of the assessment, the uncertainty that should come first is the one that gives you the best information for evaluating the other
uncertainty. Would you rather know if Joe Montana played in estimating the
49ers chances of winning or if the 49ers won in estimating whether Joe played
or not? (See the solution to problem 4.1.) As you can see from this example,
one order is usually much more natural than the other for thinking about the
problem. This natural order should be used for assessing the probabilities.
Once the information has been obtained, the order can be changed (if
necessary for the analysis) via Bayes’ Rule.

4.3 “Flipping the free” refers to reordering the nodes in a tree or part of a tree
(reversing them if there are only two nodes). Since the flipped tree represents the
same state of knowledge (uncertainties) as the original tree, the principle of
flipping a free is that the probability of any event derived from the original tree
should be the same as the probability of the same event represented by the
flipped free.

Consider the medical example on page 80. Suppose a medical expert assigns a
probability that 50 percent of the total population currently has measles, and
that given a person has measles there is a chance of 85 out of 100 that the person
has spots on his or her face. If a person does not have measles, there is a .95
chance that the person has no spots on his or her face.

What is the conditional probability that someone who has spots on his or her face
has measles? What is the conditional probability that someone who does not
have spots on his or her face has measles?

The procedure used to answer this question is exactly what Supertree does
when we change the order of nodes.

Note that an endpoint value of 1.00 was used for the Supertree input below
because we are interested only in the probabilities for this problem; the actual
endpoint value is not relevant. This tree below shows the information as given.

\[
\begin{array}{ccc}
\text{Probs StateOfHealth} & \text{Exp Val} & \text{Probs Indicator} \\
0.500 \text{ Measles} & 1.00 & 0.050 \text{ Spots} \\
0.500 \text{ NoMeasles} & 1.00 & 0.950 \text{ NoSpots}
\end{array}
\]

The next tree shows the order of nodes reversed. Spots on the face indicate
a 94.4 percent chance a person has measles and lack of spots a 13.6 percent
chance of measles. According to the information in this problem, then, spots
on the face is a fairly conclusive indicator of measles, while a lack of spots is
not as strong an indicator of no measles. This is an example of the case when
an indicator is more significant for a positive result than for a negative result.

\[
\begin{array}{ccc}
\text{Probs Indicator} & \text{Exp Val} & \text{Probs StateOfHealth} \\
0.450 \text{ Spots} & 1.00 & 0.944 \text{ Measles} \\
0.550 \text{ NoSpots} & 1.00 & 0.864 \text{ NoMeasles}
\end{array}
\]
4.4 We already know that changing the order of two chance nodes (called flipping the tree) does not change the knowledge represented by that tree. What happens if we move a chance node from the right of a decision node to the left of it? And vice versa?

In answering this question, students should recognize that the order of chance nodes relative to decision nodes in a tree represents a time ordering. An uncertainty is resolved and the information learned before any decisions to the right of it in the tree are made. Thus, when an uncertainty node is placed in front of (to the left of) a decision node in a tree, the actual event that occurs will be known to the decision-maker before the decision is made. Similarly, when an uncertainty node follows a decision node, the actual event will not be known until after the decision is made.

4.5 The radiator in your car tends to overheat, but you have not fixed it because it is still winter and cold outside. The radiator overheats only 5 percent of the days it is used in cool weather. However, it overheats 70 percent of the time in warm weather. The weather report has just predicted a 1 out of 5 chance of warm weather today.

a. Draw the influence diagram for these relationships.

The influence diagram for this simple problem contains only two uncertainties.

\[ \text{Weather} \rightarrow \text{Overheat} \]

b. What is the chance your radiator will overheat today?

The tree below shows the information as given. Note that an endpoint value of 1.0 has been used; the actual value of the endpoint is not relevant to the problem.

\[
\begin{align*}
\text{Probs Weather Exp Val} & \quad \text{Probs Overheat Exp Val} \\
.200 & \quad \text{Warm} & 1.00 & \quad .700 & \quad \text{Yes} & 1.00 \\
.800 & \quad \text{Cool} & 1.00 & \quad .300 & \quad \text{No} & 1.00 \\
.180 & \quad \text{Yes} & 1.00 & \quad .500 & \quad \text{Yes} & 1.00 \\
.820 & \quad \text{No} & 1.00 & \quad .950 & \quad \text{No} & 1.00 \\
.778 & \quad \text{Warm} & 1.00 & \quad .222 & \quad \text{Cool} & 1.00 \\
.073 & \quad \text{Warm} & 1.00 & \quad .927 & \quad \text{Cool} & 1.00 \\
\end{align*}
\]

In the tree below, the order of nodes has been reversed. The unconditional probability of overheating today, then, is .18. Students might examine the effect of summer weather on the radiator by reversing the probabilities of warm and cool.

4.6 After having had pizza delivered at 11 p.m. several times a week for a number of years, you decide that there is a 70 percent chance that a pizza with a visible amount of cheese has a visible amount of pepperoni. You also figure that the
probability that a randomly selected pizza will have visible amounts of cheese and pepperoni is .40.

a. Draw the influence diagram for these relationships.

The order of the uncertainty nodes is dictated by the data given in the problem: the given probability of visible pepperoni is conditional on visible cheese.

b. What is the probability that a randomly selected pizza has a visible amount of cheese?

While not enough information is given in this problem to draw the complete tree, enough is given to calculate the unconditional probability of having visible cheese, which is \( \frac{.4}{.7} = 57 \% \).

4.7 Using first dependent probabilities and then dependent outcomes, write down your probabilities on the temperature outside given that it is 9 a.m. or 9 p.m. Assume a .5 probability that the observation is made at either time and make your chance node on temperature have three branches.

Compare the expected temperature from each method (dependent probabilities or outcomes). How different are they? Which method enabled you to give the better estimate and why? What does this tell you about the underlying process affecting the temperature? What does it tell you about your thought process?

Answering this question should help students see that the choice of dependent probabilities or dependent outcomes varies with the type of uncertainty. In assessing the temperature at 9 a.m. and 9 p.m. on any single given day, many people will find that dependent outcomes are more natural. They often have an uncertainty in estimating temperature in general, and that uncertain range will be shifted up or down according to the time of day. This process of implicit calculation in estimating uncertainty is often captured in dependent outcomes. Usually, however, it is better to make the calculation explicit by directly estimating a potential range of temperatures and adding or subtracting the diurnal shift. In contrast, when there is no underlying calculation,
dependent probabilities are often more appropriate than dependent outcomes. An example for which dependent probabilities are natural is estimating today’s temperature given yesterday’s—the relationship between today’s temperature and yesterday’s is not as clear as the one between day and night, and so no implicit calculation is involved in the assessment. Students should be able to get a sense of the difference by examining the difference in their answers by the different methods and by thinking about the ease of assessment by each method.

4.8 An expected-value decision-maker faces the following short-term investment in a given stock:

<table>
<thead>
<tr>
<th>Decision</th>
<th>Stock Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Go Big</td>
<td>Up: .1, $200</td>
</tr>
<tr>
<td>Go Small</td>
<td>Up: .1, $30</td>
</tr>
<tr>
<td>No Go</td>
<td>$0</td>
</tr>
</tbody>
</table>

a. Draw the influence diagram. Do you have enough information to do so? Yes, there is enough information to draw the influence diagram.

![Influence Diagram](image)

b. Calculate the expected value of this decision.
As can be seen from the tree below, the preferred decision is not to buy the stock, making the expected value of the overall tree 0.

<table>
<thead>
<tr>
<th>Decision</th>
<th>Exp Val</th>
<th>Probs</th>
<th>Price</th>
<th>Exp Val</th>
</tr>
</thead>
<tbody>
<tr>
<td>Go Big</td>
<td>-25.0</td>
<td>100</td>
<td>Up</td>
<td>200.0</td>
</tr>
<tr>
<td>Go Small</td>
<td>-3.8</td>
<td>100</td>
<td>Up</td>
<td>30.0</td>
</tr>
<tr>
<td>No Go</td>
<td>0.0</td>
<td>900</td>
<td>Down</td>
<td>-7.5</td>
</tr>
</tbody>
</table>

Expected Value: 0.0
c. What is the maximum amount the decision-maker should pay for perfect information on the stock price?

Perfect information allows you to go big when the stock will go up and not buy when the stock will go down. Value of perfect information is $20 – 0 = $20.

\[
\begin{array}{c|c|c}
\text{Probs} & \text{Price} & \text{Exp Val} \\
\hline
.100 & \text{Up} & 200.0 \\
.900 & \text{Down} & 0.0 \\
\end{array}
\]

\[
\begin{array}{c|c|c}
\text{Decision} & \text{Exp Val} \\
\hline
\text{GoBig} & 200.0 \\
\text{GoSmall} & 30.0 \\
\text{NoGo} & 0.0 \\
\end{array}
\]

\[
\begin{array}{c|c|c}
\text{Probs} & \text{Price} & \text{Exp Val} \\
\hline
.100 & \text{Up} & -50.0 \\
.900 & \text{Down} & 0.0 \\
\end{array}
\]

\[
\begin{array}{c|c|c}
\text{Decision} & \text{Exp Val} \\
\hline
\text{GoBig} & -50.0 \\
\text{GoSmall} & -7.5 \\
\text{NoGo} & 0.0 \\
\end{array}
\]

\[
\begin{array}{c|c|c}
\text{Probs} & \text{Exp Val} \\
\hline
.180 & \text{"Up"} & 75.0 \\
.820 & \text{"Down"} & 0.0 \\
\end{array}
\]

\[
\begin{array}{c|c|c}
\text{Decision} & \text{Exp Val} \\
\hline
\text{GoBig} & 75.0 \\
\text{GoSmall} & 11.3 \\
\text{NoGo} & 0.0 \\
\end{array}
\]

\[
\begin{array}{c|c|c}
\text{Probs} & \text{Price} & \text{Exp Val} \\
\hline
.012 & \text{Up} & 200.0 \\
.988 & \text{Down} & -50.0 \\
\end{array}
\]

\[
\begin{array}{c|c|c}
\text{Decision} & \text{Exp Val} \\
\hline
\text{GoBig} & -47.0 \\
\text{GoSmall} & -7.0 \\
\text{NoGo} & 0.0 \\
\end{array}
\]

d. Suppose there is a test that can predict the stock price with an accuracy of .9. Draw the influence diagram for this. What is the maximum amount the decision-maker should pay for this test?

The influence diagram shows that the predictor (indicator) of stock price is influenced by the actual stock price (state of nature). The predictor influences the decision, thus providing the decision-maker with information before the decision is made.

The tree below shows the value with imperfect information. Note that the accuracy of the test results in a 50/50 chance the stock will go up or down when “up” is predicted. The value of imperfect information is $13.50 – 0 = $13.50.

The tree below verifies that the test information was input correctly in the order Nature’s tree (nature first, then the predictor). To obtain this display, we have reordered the tree with the Prediction node following the Price node. Note that the No Go option is followed by the Price and Prediction nodes (with
somewhat strange looking probabilities) because of Supertree’s automatic symmetrization in drawing the tree.

e. Suppose the test says the stock will rise and this information is given to the decision-maker for free. Now what is the value of perfect information on the stock price?

As shown below, the imperfect accuracy of the predictor makes perfect information on the stock price still valuable, but less so. Given that the decision-maker already has imperfect information, the value of perfect information is $20 - 13.50 = $6.50.

f. Suppose a wizard comes along. He can make any possible outcome happen the decision-maker desires. Draw the influence diagram for the value with the wizard. What is the maximum amount the wizard should be paid?

This influence diagram is similar to the one for perfect information except that the Stock Price uncertainty has a new probability distribution—the distribution given wizardry.
As you can see in the perfect information tree in problem 4.8b, the best possible value is $200. Accordingly, value of control is $200 – 0 = $200.

4.9 **Ursa Major Movies (UMM) has been trying a blind test on all its movies before releasing them. The test labels a movie as a “Hit” or a “Dud.” To make the test blind, UMM released all movies regardless of the test result. The test result and the actual history of the movies are shown below.**

<table>
<thead>
<tr>
<th>Test Result</th>
<th>&quot;Hit&quot;</th>
<th>&quot;Dud&quot;</th>
</tr>
</thead>
<tbody>
<tr>
<td>Broke box office records</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>Run of the mill</td>
<td>13</td>
<td>7</td>
</tr>
<tr>
<td>Disaster</td>
<td>8</td>
<td>9</td>
</tr>
</tbody>
</table>

a. **Draw the influence diagram for the test and movie results. Is it in the order of Nature’s tree? Why or why not?**

The table shows the track record of the test and provides the information for the likelihood function—the probability of Test Results given Movie History. The proper influence diagram is in the order of Nature’s tree, as shown below. The table will provide the probabilities for the Test Result node.

b. **What is the probability that a film chosen at random out of the studio’s past movies was a disaster? Run of the mill? Broke records?**

The probabilities are given in the right column of the table below. This information might be of use in obtaining a probability for whether the next movie will be a success or not (the prior probability). However, beware of two traps. First, do not let the data overwhelm your common sense—you may have some information about the next movie that renders your state of information more than “random.” Second, this type of data is often biased—make sure that the test was truly blind and that all movies in the test were released regardless of the test result.

<table>
<thead>
<tr>
<th>Test Result</th>
<th>&quot;Hit&quot;</th>
<th>&quot;Dud&quot;</th>
<th>Total</th>
<th>Odds</th>
</tr>
</thead>
<tbody>
<tr>
<td>Broke box office records</td>
<td>5</td>
<td>1</td>
<td>6</td>
<td>6/43</td>
</tr>
<tr>
<td>Run of the mill</td>
<td>13</td>
<td>7</td>
<td>20</td>
<td>20/43</td>
</tr>
<tr>
<td>Disaster</td>
<td>8</td>
<td>9</td>
<td>17</td>
<td>17/43</td>
</tr>
<tr>
<td>Totals</td>
<td>26</td>
<td>17</td>
<td>43</td>
<td></td>
</tr>
</tbody>
</table>
c. What is the probability that a disaster had previously been labeled a “Hit”? What is the probability that a box office record breaker had been labeled a “Dud”? Is the test better at detecting good or bad movies?

The probabilities are: disaster labeled a “Hit”—8/17; record breaker labeled a “Dud”—1/6. The test is better at identifying good movies than bad ones. These are probabilities that are part of the likelihood function and that will be input into Nature’s tree.

d. The producer thinks that a new movie is really quite good (a 5 in 10 chance of being a box office hit, a 3 in 10 chance of being run of the mill). After learning that the test came up “Dud,” how should the producer revise her probabilities?

These are the prior probabilities that will be input into Nature’s tree. The first tree below shows the information as given. The probabilities for Test are, of course, 5/6, 1/6, 13/20, 7/20, 8/17, and 9/17 put in decimal form. The second, reversed tree shows the probabilities for the movie outcome given the test results. Since the test is fairly good at detecting record breakers (5/6 identification rate), the producer should reduce her probability of the movie’s being a hit and increase her probability of it being run of the mill. The tree shows that the probability for a box office hit is now reduced from 50 percent to 28 percent, and for a run of the mill increased from 30 percent to 36 percent. The probability of a disaster has increased from 20 percent to 36 percent.

e. The president thinks that this new movie is like all the others, meaning that the historical frequencies above apply. What should he think after learning that the test result for this movie was “Dud”? 

He should think that there is a 1/17 chance the movie will be a record breaker, a 7/17 chance it will be run of the mill, and a 9/17 chance it will be a disaster. These results are derived by using both the prior and the likelihood information contained in the table in problem 4.9b.
f. Assume a record breaker gives the company a net profit of $20 million; a run of the mill, $2 million; and a disaster, a net loss of $2 million. What value would the producer place on the new film before learning the test results? After learning the test results? How about the president?

The tree below has the values for Broke Records, Run of the Mill, and Disaster. Because the test does not affect the value and because this tree has the producer’s probabilities, the expected value is the producer’s value before the test, or $10.2 million.

The value of the movie for the producer after the test is shown below: it is $12.09 million if Hit is predicted and $5.66 million if Dud is predicted.

A similar tree can be constructed for the president by using probabilities from the table in problem 4.9b. By historical measures, the president would value the movie at $2.93 million. If a Hit was predicted, this value would rise to $4.23 million, and if a Dud was predicted, the value would fall to $0.94 million.

C. Thompson, the credit manager of IJK Industrial Products, considered extending a line of credit to Lastco Construction Company. Lastco was a new company and was definitely considered a credit risk. Drawing on his experience, Thompson said, “There is about a 30 percent chance Lastco will fail within the year, which means a severe credit loss. And the way these construction companies operate, I would say there is another 25 percent chance Lastco will run into serious financial trouble.” After being further questioned about other
possibilities, Thompson said. “If they don’t run into financial problems, there still is less than a 50/50 chance of Lastco becoming a regular customer. I would say the odds are about 5 to 4 that Lastco will end up being a sporadic customer.” Thompson also made the following predictions:

- If Lastco failed completely, it would average purchases of $1,500 before failing but leave an average unpaid balance of $800, which would be totally lost.
- If Lastco had severe financial troubles, it would lose its credit but only after purchases of $2,000, including an unpaid balance of $1,000, of which $500 would ultimately be collected.
- As a sporadic customer, Lastco would average purchases of only $500 (with no credit losses). However, as a good customer, it would average purchases of approximately $6,000.

IJK was concerned about granting credit to Lastco. On the one hand, if it did not extend credit to a potential customer, business was lost. On the other hand, there was a substantial risk of nonpayment (as described above), and since IJK made an average profit (price minus variable cost) of only 20 percent of sales, this exacerbated the problem. In addition, there were collection costs of $100 per customer for those that failed or were in financial trouble.

a. **Draw the influence diagram for this case.**

One way to structure the problem is shown in the diagram below.

![Influence Diagram](image)

b. **Construct the decision tree for this case.**

The tree can be structured using the Treevalue option for the endpoint.

<table>
<thead>
<tr>
<th>STRUCTURE</th>
<th>NAMES</th>
<th>OUTCOMES</th>
<th>PROBABILITIES</th>
</tr>
</thead>
<tbody>
<tr>
<td>1D2 4</td>
<td>Credit</td>
<td>Grant Deny</td>
<td></td>
</tr>
<tr>
<td>2C3 3 3 3</td>
<td>Lastco</td>
<td>Fail Trouble Sporadic &amp; Regular</td>
<td>.3 .25 .25 .2</td>
</tr>
<tr>
<td>3E</td>
<td>T$Profit</td>
<td>Depends on 2</td>
<td></td>
</tr>
<tr>
<td>4E</td>
<td>0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Lastco</th>
<th>VALUE, NODE 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fall</td>
<td>-600</td>
</tr>
<tr>
<td>Trouble</td>
<td>-200</td>
</tr>
<tr>
<td>Sporadic</td>
<td>100</td>
</tr>
<tr>
<td>Regular</td>
<td>1200</td>
</tr>
</tbody>
</table>
c. Should IJK grant credit to Lastco?

As shown in the tree below, granting credit is the preferred option. It has an expected value of $35, versus the expected value of 0 for not granting credit.

\[
\begin{array}{c|c|c}
\text{Probs} & \text{Exp Val} & \text{Expected Value: 35.00} \\
\hline
\text{Lastco} & \text{Grant} & 35.00 \\
\text{Deny} & 0.00 & \\
\end{array}
\]

\[
\begin{array}{c|c|c|c}
\text{Credit Exp Val} & \text{Probs Lastco} & \text{Exp Val} \\
\hline
\text{Grant} & 0.300 \text{ Fail} & -600.00 \\
\text{Deny} & & 0.00 \\
\text{Grant} & 0.250 \text{ Trouble} & -200.00 \\
\text{Deny} & & 0.00 \\
\text{Grant} & 0.250 \text{ Sporadic} & 100.00 \\
\text{Deny} & & 0.00 \\
\text{Grant} & 0.200 \text{ Regular} & 1200.00 \\
\text{Deny} & & 0.00 \\
\end{array}
\]

d. Suppose a credit rating company could somehow provide perfect information on a potential customer for $200. Should IJK buy it?

The value of perfect information is $265 – 35 = $230, making the perfect credit information worthwhile because it costs only $200.

\[
\begin{array}{c|c|c|c|c|c|c}
\text{Probs} & \text{Exp Val} & \text{Credit Exp Val} & \text{Expected Value: 265.00} \\
\hline
\text{Lastco} & \text{Grant} & -600.00 & & & \\
\text{Deny} & 0.00 & & & & \\
\text{Grant} & 0.300 \text{ Fail} & 0.00 & & & \\
\text{Deny} & & 0.00 & & & \\
\text{Grant} & 0.250 \text{ Trouble} & 0.00 & & & \\
\text{Deny} & & 0.00 & & & \\
\text{Grant} & 0.250 \text{ Sporadic} & 100.00 & & & \\
\text{Deny} & & 0.00 & & & \\
\text{Grant} & 0.200 \text{ Regular} & 1200.00 & & & \\
\text{Deny} & & 0.00 & & & \\
\end{array}
\]

e. Suppose the fee of the credit rating company was only $50, but the company could provide only “good opinions” (not perfect information) about potential customers. Suppose also that Thompson has some experience with credit rating companies, which he says applies to the Lastco decision. His experience with ratings is summarized below as credit ratings by customer classification (percent of total.)

\[
\begin{array}{c|c|c|c|c}
\text{Rating} & \text{Failed} & \text{Financial Troubles} & \text{Sporadic Customer} & \text{Good Customer} \\
\hline
\text{Good} & 0 & 10 & 40 & 40 \\
\text{Medium} & 40 & 50 & 50 & 50 \\
\text{Poor} & 100 & 100 & 100 & 100 \\
\end{array}
\]

Note: The table should be interpreted as follows: For example, in similar situations of companies that failed, none had been rated good, 40 percent had been rated medium, and 60 percent had been rated bad.

Would it be worthwhile to use the credit rating company? Illustrate your answer with a revised influence diagram.

The influence diagram is shown below. Note that this is the full influence diagram with the decision on whether to purchase the credit information included explicitly.
The tree can be put into Supertree in compact form by recognizing that a node can have probabilities that depend on a node later in the tree. Note that the cost of the credit check has been put in as a reward of –50.

As can be seen below, even with the credit company fee of $50, obtaining the credit check is the preferred alternative, with an expected value of $86.50; this is better than the $35 expected value from granting credit without a credit check.
4.11 Most market surveys give imperfect information. The example below shows a symmetric situation—a fraction, \( q \), of product successes had positive survey results, and a fraction, \( q \), of failures had negative survey results. The prior probability of a product success is given as \( p \).
a. Flip the tree and calculate the posterior probabilities for Product Outcome and the preposterior probabilities for Survey Result in terms of \( p \) and \( q \).

\[
\begin{align*}
\text{Survey Result} & \quad \text{Product Outcome} \\
\text{Success} & \quad \text{Success} \\
\text{Positive} & \quad \frac{pq(pq+(1-p)(1-q))}{(1-p)(1-q)(pq+(1-p)(1-q))} \\
\text{Negative} & \quad \frac{p(1-q)+(1-p)q}{p(1-q)+(1-p)q} \\
\text{Failure} & \quad \frac{(1-p)(1-q)}{pq+(1-p)(1-q)} \\
\end{align*}
\]

b. Plot the probability of a product success given a positive survey result against the value of \( p \). Do this for \( q = .5, .6, .7, .8, .9, \) and 1.0. Why are values of \( q \) less than .5 and greater than 1 not needed?

Values of \( q \) greater than 1 are not needed because probabilities are not greater than 1, and \( q \) is a probability. Values of \( q \) less than .5 are not needed because, for probabilities less than .5, the indicator is wrong more than it is right, and decisions would be made by reinterpreting “negative result” as “positive result” and using the probability \( 1 - q \).

\[ p(\text{success|positive,p,q,}S) \]

c. How useful is the survey result for values of \( p \) near 0, .5, and 1.0? How does this depend on the value of \( q \)? Explain qualitatively what this means in terms of uncertainty and certainty and the accuracy of surveys.

For prior probabilities near the value \( p = 1 \), we expect success, and the positive survey result serves only to reinforce our prior estimate of success. Near \( p = 1 \), the exact value of \( q \) is not important—the curves for different
values of $q$ are close together, and the probability of success (vertical axis) for a given value of $p$ near 1 is almost the same for any value of $q$. For instance, for $p = .9$, $p(\text{success} \mid \text{positive}, p=.9,q,S)$ is .99, .97, .95, .93, and .90 for $q$ equal to .9, .8, .7, .6, and .5, respectively. The positive survey result has increased our probability of success a little.

To examine the possibility that they survey result contradicts our prior expectations, consider prior probabilities very near $p = 0$. Here we expect failure, and the positive survey result contradicts our expectations. If $p$ is very near 0, we are so certain of failure that the effect of the (imperfect) information from the survey is small. As we move away from $p = 0$, we see the important contribution of the survey. For instance, for $p = .1$, $p(\text{success} \mid \text{positive}, p=.1,q,S)$ is .50, .31, .21, .14, and .10 for $q$ equal to .9, .8, .7, .6, and .5, respectively. Positive results for a reliable test (e.g., $q = .9$) have a dramatic effect on our probability of success.

Around $p = .5$, we see fairly dramatic results, but the biggest effect of a positive survey result occurs for values of $p$ between .2 and .4. Here prior expectations of failure were not very strong, and the new information from the survey has a strong effect.

Note that there is a big difference between a survey with $q = .9$ and one with $q = 1!$ Note also the result that a survey with $q = .5$ has no value—$p(\text{success} \mid \text{positive}, p,q=.5,S) = p$. 

5

Attitudes Toward Risk Taking

Problems and Discussion Topics

5.1 Consider several decisions you have made, ranging from minor importance to major importance. Was there implicit or explicit risk aversion in the way you went about making these decisions? Do you think your risk attitude was consistent across these decisions? Give examples and say why or why not.

Indications of risk aversion in a decision include a particular desire to avoid a certain outcome that seems stronger than warranted by its probability of occurring. This desire can be manifested, for instance, in an unwillingness to consider certain alternatives because they might lead to the undesired outcome. For example, a person might not even want to consider going on a picnic when the weather forecast includes a 10 percent chance of rain. An example of inconsistency might be that same person not thinking twice about going to a baseball game the same day—though this might reflect varying valuations of being in each place in the rain, rather than apparently different amounts of aversion to the risk of being caught in the rain.

The way people gamble and make investments usually provides a simpler example of varying (and inconsistent) risk aversion. Many people are willing to invest $1 in a slot machine, which has a negative expected value (thus indicating risk-seeking behavior), while only tolerating some small possibility of losses in stock investments (thus indicating mild risk aversion). He or she might also have zero deductible in an insurance policy (thus indicating strong risk aversion). See the next question for discussion of reasons why this might be so.

5.2 Do the behavioral axioms adequately describe the way you would like to make certain types of decisions? Are there cases where they do not fit the way you would like to make a decision? If so, give an example.

As mentioned above, a number of common behaviors do not fit the behavioral axioms. For instance, people may be willing to choose negative expected
value alternatives (as in gambling), while demanding positive expected values in other aspects of life.

Mortgages with fixed or variable interest rates are an area with great potential for violating the behavioral axioms. Often, when competing fixed and variable interest mortgages are considered side by side, there are very few (and often improbable) interest rate scenarios under which the fixed rate plan would be less expensive than competing variable rate plans. However, people commonly prefer the fixed rate plan nonetheless, sometimes manifesting almost a refusal to establish a certain equivalent for an uncertain venture (violating axiom 2).

It is, of course, a separate question to consider whether these behaviors describe the way people would like to make decisions. It is often true that people change the way they make decisions after examining how they go about it— awareness leads to change. The economic implications of risk aversion (embodied in the risk penalty) may lead people to desire less risk aversion (or even no risk aversion) for particular situations that they formerly approached with greater risk aversion.

5.3 What factors contribute to the difference between using the expected value and using a utility function with risk aversion? Under what circumstances would the expected value and the certain equivalent value be the same?

The rule of thumb here is that the greater the differences between the potential outcomes become (larger spread of possibilities) and the larger the positive or negative outcomes become (higher stakes), the more the certain equivalent value and the expected value diverge. This effect stems from the mathematics of computing expected utility, from the greater curvature of risk-averse utility functions in the region of large negative values (greater overvaluing), and from the lesser curvature for large positive values (greater undervaluing).

The expected value and the certain equivalent value would be the same where there is no uncertainty or where the stakes are “small” (as indicated by the utility curve being nearly linear over the range of values).

5.4 Suppose you have a certain equivalent CE for a venture with probabilities \(p_1, p_2, \ldots, p_n\) and prizes \(x_1, x_2, \ldots, x_n\). The Delta Property states that if we add some arbitrary amount \(D\) to all the prizes such that the venture is now for prizes \(x_1 + D, x_2 + D, \ldots, x_n + D\), then your certain equivalent for the new venture will be \(CE + D\). Furthermore, if you subscribe to the Delta Property, then your utility function is exponential or linear.

Describe a situation where the Delta Property would not apply to you.

Most situations where the Delta Property would not apply involve one of two possible sets of circumstances.

The first occurs when the person or company needs a particular sum of money and anything less than it will not do. For instance, if a margin call on your stocks comes due tomorrow and you need $50,000 more than you have in cash right now and you have a choice between the two ventures below
(with immediate payoffs), you might prefer a .8/.2 chance at $20,000 or $0 ($16,000 expected value) over a .3/.7 chance at $40,000 or $0 ($12,000 expected value). Neither venture will bring the needed $50,000.

However, if $10,000 were added to all the possibilities (changing the respective expected values to $26,000 and $22,000, as shown below), you might prefer the second opportunity, because there is a chance of making $50,000—the amount you need. This behavior violates the Delta property, which states that your choice should be the same as it was above.

The second possible (and more common) situation is that people’s behavior changes when their total assets change. For instance, given a .8/.2 chance at $20,000 or –$10,000 ($14,000 expected value) and a .5/.5 chance at $20,000 or $0 ($10,000 expected value), most people would take the second alternative.

However, if $30,000 were added to each outcome (making the possible outcomes $50,000 or $20,000, and $50,000 or $30,000) as shown below, many people would take the .8 chance of winning $50,000 over the .5 chance of winning $50,000. This change could be explained by the outcomes being large enough to change a person’s asset levels. Another way to look at it is to say that $30,000 has been added to your assets and now the prospects of loss in the venture on the left above is less intimidating. T. Boone Pickens might have taken that alternative both times. What would you do?
5.5 Use one of the two methods described in the text to assess a risk tolerance for yourself. Have the rewards or losses used in this assessment be paid immediately.

Now consider that the money you invest or lose can be paid monthly over a thirty-year period. For instance, at 10 percent you would pay roughly $1,000 each month for the next thirty years to pay off $100,000. Reassess your risk tolerance. Is it any different? Why or why not? What if interest were included in the balance?

This question highlights the problem of asset level in assessing a utility function. As mentioned in the discussion for problem 5.4, the amount of money people have in the bank often affects the amount they are willing to risk having to pay today. Spreading the payments out over a long time allows people to compare possible payments or losses against their expected earnings over that period rather than against their present asset level. (Consider the usually large difference between the two sums.) Including an interest rate adds the wrinkle of the person’s time value of money and how he or she expects their earnings to change over the period in question.

5.6 Use the method described in the text to encode a utility function for a classmate. Then, directly assess his or her certain equivalent (minimum selling price) for a 1 in 5,000 chance of winning $10,000. Compare the result with the certain equivalent from using the utility function. What does this tell you about your classmate’s risk attitude for this kind of opportunity?

Most encodings will produce a risk-averse utility function. Directly assessing a certain equivalent on the 1 in 5,000 chance of $10,000 will often produce a certain equivalent higher than the expected value—indicating a risk-seeking attitude for this situation. This contradictory but common situation of risk aversion overall and risk seeking for small-probability, high-payoff gambles can be explained behaviorally by noting that the potential payoffs are, for most people, unachievable otherwise: there are few chances for instant wealth. This possibility of a sudden windfall is enough to keep Las Vegas and the state lotteries in business.

5.7 Peter Portfolio faces a decision for a short-term investment based on the prospective movement of a stock. The possibilities are shown below.
Peter has an exponential utility function with a risk tolerance of $5,000.

a. What is Peter’s decision? What is his certain equivalent for the venture?
As the tree below shows, Peter’s preferred decision is to buy the stock, with a certain equivalent of $278.

b. Peter is not sure if his risk tolerance is exactly $5,000. For what range of risk tolerance should he “Buy”?
In the risk sensitivity plot below, the certain equivalents of the two alternatives (Buy and Stay Out) are displayed for varying risk tolerances (the bottom scale). For risk tolerances greater than about $2,100, Peter should buy the stock.
c. Sheba Sisters brokerage firm has investigated the stock and offers Peter perfect information on whether the stock price will go up or down. What is the maximum he should pay for the information?

The tree drawing below shows the certain equivalent with perfect information. The value of perfect information is $901 - 278 = $623. This amount is the maximum Peter should pay for information on the stock.

5.8 Your regular morning radio show has awarded you the uncertain venture shown below.

You have an exponential utility function with a risk tolerance of $2,000. You are indifferent to selling the lottery for $700. What is the probability $p$?
The probability is calculated as shown below.

\[ u(700) = pu(1000) + (1 - p)u(-500) \]

\[ u(x) = -e^{-x/R} \]

\[ u(700) = -e^{700/2000} = -.7047 \]
\[ u(1000) = -e^{1000/2000} = -.6065 \]
\[ u(-500) = -e^{500/2000} = -1.2840 \]

\[ -.7047 = p(-.6065) + (1 - p)(-1.2840) \]

\[ .5793 = .6775p \]

\[ p = .8551 \]

This result can be confirmed by entering this value of \( p \) into Supertree and displaying the tree, as shown below.

5.9 J. K. Kay faces the following short-term investment decision on stocks A and B whose performance is correlated.

<table>
<thead>
<tr>
<th>Decision</th>
<th>Stock A</th>
<th>Stock B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Invest</td>
<td>Up</td>
<td>$100</td>
</tr>
<tr>
<td></td>
<td>Up</td>
<td>$100</td>
</tr>
<tr>
<td></td>
<td>2/5</td>
<td>$100</td>
</tr>
<tr>
<td></td>
<td>1/2</td>
<td>$100</td>
</tr>
<tr>
<td></td>
<td>3/5</td>
<td>$100</td>
</tr>
<tr>
<td></td>
<td>Down</td>
<td>$100</td>
</tr>
<tr>
<td></td>
<td>4/5</td>
<td>$100</td>
</tr>
<tr>
<td></td>
<td>Pass</td>
<td>$100</td>
</tr>
<tr>
<td></td>
<td>Down</td>
<td>$100</td>
</tr>
<tr>
<td></td>
<td>1/5</td>
<td>$100</td>
</tr>
<tr>
<td></td>
<td>Up</td>
<td>$100</td>
</tr>
<tr>
<td></td>
<td>1/2</td>
<td>$100</td>
</tr>
<tr>
<td></td>
<td>3/5</td>
<td>$100</td>
</tr>
<tr>
<td></td>
<td>Down</td>
<td>$100</td>
</tr>
</tbody>
</table>

a. If JK is an expected-value decision-maker, how much should he pay the clairvoyant for perfect information on whether stock B goes up or down?
The initial tree for this problem is shown below.

![Decision Tree Image]

From the tree display with the stock B node brought to the front (below), the value of information is $25 - 0 = $25.

![Decision Tree Image]

b. If JK is risk averse and has an exponential utility function with a risk tolerance of $500, what is the most he should pay for the perfect information on stock B?

From the initial tree with a risk tolerance of $500 (below) the certain equivalent for buying the stock has become $–13.10, making the no investment value of zero preferred.

![Decision Tree Image]

From the reordered tree (below), the value of perfect information with risk aversion is $23.4 - 0 = $23.40, slightly less than the value of information on an expected-value basis.

![Decision Tree Image]

5.10 Compare the certain equivalents from using the exponential utility function for some problems in previous chapters with the certain equivalents obtained from
the following approximation:

\[
\text{Certain Equivalent} = \text{Expected Value} - \frac{1}{2} \frac{\text{Variance}}{\text{Risk Tolerance}}
\]

where

\[
\text{Expected Value} = \sum_{i=1}^{m} x_i p(x_i \mid S)
\]

\[
\text{Variance} = \sum_{i=1}^{m} (x_i - \text{Expected Value})^2 p(x_i \mid S)
\]

Many answers are possible, depending on what risk tolerances are used. We therefore suggest that in assigning this question, you indicate which risk tolerances to use in each specific problem. It is then a simple process to set the risk tolerance in Supertree and use the Rollback Tree command to compute the overall certain equivalent for the tree and the List Distribution Values command to obtain the expected values and variances.

5.11 Je has the following utility function:

\[
u(x) = \ln(1 + x / R)
\]

where \( R = \$200 \). His utility is set so that outcomes \( x \) are measured as differences from his present wealth. There is an uncertain venture, \( L \), shown below.

Assume that Je can borrow money at no cost.

a. Suppose that Je does not own the venture. What is the maximum he should be willing to pay for this venture?

This problem produces rather strange results because of the utility function used and the expected value of zero. Instructors might also want to assign the problem with possible outcomes of $300 and –$100 (have the students do this one first), so students can see more intuitive results before trying to follow through the implications of the problem as stated. With a buying price of \( B \),
36. **ANSWERS TO PROBLEMS AND DISCUSSION NOTES**

\[ 0 = u^{-1}[5u(100 - B) + 0.5u(-100 - B)] \]
\[ u(0) = 0.5\left[u(100 - B) + u(-100 - B)\right] \]
\[ \ln(1 + 0 / R) = 0.5\left[u(100 - B) + u(-100 - B)\right] \]
\[ 0 = 0.5\left[u(100 - B) + u(-100 - B)\right] \]
\[ 0 = u(100 - B) + u(-100 - B) \]
\[-u(100 - B) = u(-100 - B) \]
\[-\ln[1 + (100 - B) / R] = \ln[1 + (-100 - B) / R] \]
\[-\ln[(R + 100 - B) / R] = \ln[(R - 100 - B) / R] \]
\[ \ln[R /(R + 100 - B)] = \ln[(R - 100 - B) / R] \]
\[ R / (R + 100 - B) = (R - 100 - B) / R \]
\[ R^2 = (R + 100 - B)(R - 100 - B) \]
\[ 200^2 = (200 + 100 - B)(200 - 100 - B) \]
\[ 40,000 = (300 - B)(100 - B) \]
\[ 40,000 = 30,000 - 100B + B^2 \]
\[ 0 = B^2 - 400B - 10,000 \]

\[ B = \frac{-(-400) \pm \sqrt{(-400)^2 - 4(1)(-10,000)}}{2(1)} \]
\[ B = \frac{400 \pm \sqrt{160,000 - 40,000}}{2} \]
\[ B = \frac{1}{2}(400 \pm 447) \]
\[ B = \frac{1}{2}(400 \pm 447) \]
\[ B = -23.50, 423.50 \]

Ignoring the larger root, Je’s buying price is –$23.50, meaning that someone would have to pay him $23.50 to get him to accept the lottery.

**b. Assume that he owns the venture. What is the lowest price that he should be willing to sell it for?**

The selling price can also be calculated.

\[ selling \ price = s = u^{-1}[5u(100) + 0.5u(-100)] \]
\[ s = u^{-1}[5(4.055) + 0.5(-0.6931)] \]
\[ s = u^{-1}(-0.1438) \]
As indicated by the above result, given his utility function and risk tolerance, Je would actually be willing to pay someone $26.79 to take this lottery off his hands.

5.12 There are extreme situations in which people may behave differently than normal. Some of these situations can be explained by a special utility function.

Mr. Sam Spade, after a night of partying in San Francisco, suddenly realizes that he has only a $10 bill left in his pocket. Unfortunately, the train fare home for him is $15. Then he observes a wild gambler in the nearby corner who offers him the three opportunities shown below, each at a certain cost:

- **L1:**
  - Cost: $8
  - 1/2 chance of $48, 1/2 chance of $0

- **L2:**
  - Cost: $6
  - 1/3 chance of $21, 2/3 chance of $0

- **L3:**
  - Cost: $10
  - 1/4 chance of $200, 3/4 chance of $0

Right now, all Sam cares about is getting home. What is his special utility function for this situation? Which opportunity should Sam choose?

Because train fare is $15, Sam does not care about any payoff less than $15, and any extra payoff is similarly worthless. So he has the following utility function:

\[ u(x) = \begin{cases} 
0 & \text{if } x < 15 \\
1 & \text{if } x \geq 15 
\end{cases} \]

Accordingly, Sam’s utility for the first lottery is .5, his utility for the second one is .33, and his utility for the third is .25. Sam should therefore pick the first lottery because it has the highest utility. The cost is irrelevant, because he has enough money to buy any one of the lotteries.

5.13 Missouri Tubing, Inc., is a manufacturer of specialty steel and copper tubing. Its production facility is located adjacent to the Missouri River and is protected from the waters by a 20-foot-high dike. Extremely heavy rains in recent weeks have raised the level of the river dangerously close to the top of the dike. Several other areas near the river have already been badly flooded and the weather forecast is for continued rain.

The risk manager of Missouri Tubing estimates that there is a 20 percent chance that the river will top the dike in the coming weeks and flood the factory. If the factory is flooded, there is a 50 percent chance the damage will be heavy,
costing about $20 million to repair, and a 50 percent chance it will be light, costing about $10 million. Furthermore, flooding of the factory will force it to shut down while repairs are made. If damage is heavy, the factory will be closed for four months. If damage is light, there is a 60 percent chance the shutdown will last four months and a 40 percent chance it will last only two months. For each month the factory is closed, Missouri Tubing will lose $25 million in profits.

The risk manager now regrets that he recently cancelled the company’s insurance policy covering flood damage. However, his insurance agent has decided to help him out by offering him a special emergency flood insurance policy. The policy provides coverage of both property damage and business interruption (i.e., lost profits) from flooding for a period of six months. The premium for the policy is $30 million. Another policy providing only the business interruption coverage is also available for a premium of $25 million.

a. Draw the influence diagram for Missouri Tubing’s problem.

b. Structure the decision tree for Missouri Tubing’s problem and calculate the expected value of each alternative.

The display below shows one way of structuring this tree. A node for the insurance decision is followed by chance nodes for whether or not the river floods, for the damage if there is a flood, and for the lost profits. The outcomes of node 1 are coded as “1” for no insurance, “2” for lost profits insurance only, and “3” for lost profits and damage coverage. The costs of the insurance policies have been included as rewards for node 1. The endpoint node looks at the insurance policy selected and adds in damage if full coverage (“3”) was not chosen and lost profits if no coverage was chosen (“1”). Note also that the probabilities for lost profits depend on the level of damage from flooding.
The tree drawing below shows that purchasing no insurance is the preferred alternative, with expected total losses of $21 million versus $28 million for lost profits coverage and $30 million for full insurance coverage.

**c. Calculate the value of clairvoyance for an expected-value decision-maker on whether or not the Missouri Tubing factory is flooded.**

From the partial tree drawing below, the value of perfect information on whether the factory is flooded is \((-6) - (-21) = \$15\) million.

**d. Calculate the certain equivalent for each alternative assuming that Missouri Tubing’s risk tolerance is \$50\) million.
With risk aversion, the lost profits coverage (alternative "2") becomes preferred, with a certain equivalent of \(-$28\) million versus \(-$47\) million for no coverage and \(-$30\) million for full coverage.

<table>
<thead>
<tr>
<th>Certain Equivalent: -28.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Insure Cert Eq Prob River Cert Eq</td>
</tr>
<tr>
<td>1 109.2 47.3</td>
</tr>
<tr>
<td>2 -100 10.0</td>
</tr>
<tr>
<td>&gt;2 -28.4</td>
</tr>
<tr>
<td>3 -30.0</td>
</tr>
</tbody>
</table>

\(e.\) Calculate the value of clairvoyance with risk aversion on whether the Missouri Tubing factory is flooded.

Value of information with risk aversion is \((-7.61) - (-28.44) = $20.8\) million, an increase of $5.8 million from the value of information without risk aversion. How much should Missouri Tubing be willing to spend to build a levee around the factory? (What is the value of control with risk aversion on flooding?)

\(5.14\) Suppose that a decision-maker agrees on the five utility rules. We then know that there exists a utility function for the decision-maker. If he or she also agrees on the Delta Property, then we know further that the utility function is exponential and can be characterized by a single number, the risk tolerance. (See problems 5.4 and 5.17.)

One way to approximate the risk tolerance is to find the largest number \(X\) for which the venture shown below is still acceptable to the decision-maker. Show that \(X\) is within about 4 percent of the true risk tolerance.

\[ \begin{array}{c}
   \text{Insure Cert Eq} \\
   1 -109.21 \\
   2 -40.25 \\
   3 0.00 \\
\end{array} \]

If the venture is just still acceptable, then it has a certain equivalent of zero.
Putting this into the calculation of the utility of the venture,

\[ u(0) = 0.5u(X) + 0.5u(-X / 2) \]

and letting

\[ u(y) = -e^{-y/R} \]

(an exponential utility function with parameters \( a = 0 \) and \( b = 1 \)),

\[ -1 = -0.5e^{-X/R} - 0.5e^{X/2R} \]
\[ 2 = e^{-X/R} + e^{X/2R} \]
\[ 0 = \left(e^{-X/2R}\right)^3 - 2\left(e^{-X/2R}\right)^2 + 1 \]

You could solve this equation as a cubic equation, or you can solve it numerically—i.e., plug in different values of \( X/R \). The equation is satisfied by \( X/R = 0.96 \), meaning \( X \) is within 4 percent of the risk tolerance. The equation is also solved by \( X/R = 0 \). This value is explained in the solution to problem 5.15.

5.15 Show that the largest value \( X \) for which the venture shown below is acceptable is within 10 percent of the risk tolerance (assuming an exponential utility function).

As in the previous problem, use the exponential utility function \( u(y) = -e^{-(y/R)} \).

\[ u(0) = 0.75u(X) + 0.25u(-X) \]
\[ -e^{-0/R} = -0.75e^{-X/R} - 0.25e^{X/R} \]
\[ -1 = -0.75e^{-X/R} - 0.25e^{X/R} \]
\[ \left(e^{X/R}\right)^2 - 4e^{X/R} + 3 = 0 \]
\[ e^{X/R} = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(3)}}{2(1)} \]
\[ e^{X/R} = 3, 1 \]
\[ X / R = \ln(3), \ln(1) \]
\[ X / R = 1.099, 0 \]

The first result shows the desired result—that \( X \) is within 10 percent of the risk tolerance. The second root shows that the certain equivalent is also
Thus, for a .75/.25 chance at X or −X, the certain equivalent is graphed below.

At X = 0, the venture is worth nothing. When 0 < X < 1.099R, the venture is attractive to the decision-maker. When X > 1.099R, the certain equivalent becomes negative, because the stakes have become too large for the decision-maker. For this venture, the most attractive opportunity occurs when X/R is around .55.

5.16 In general, the risk tolerance function can be defined as

\[ \rho(x) = -u'(x)/u''(x) \]

where x is the increment to current wealth of the person in question (wealth = x + current wealth), u' is the first derivative of the utility function, and u'' is the second derivative. Some people agree that “If I had more wealth, then I would probably tolerate more risks.” To a first approximation, this can be captured by:

\[ \rho(x) = \rho_0 + Kx \] where \( K \geq 0 \)

which says that risk tolerance would increase if wealth increased. At the current level of wealth, x = 0 and the risk tolerance is just \( \rho_0 \). What are the values of K and \( \rho_0 \) for the following utility functions?
a. $u(x) = a - b^{x/R}$

This is, of course, the exponential utility function.

$$u'(x) = \frac{b}{R} e^{-x/R}$$

$$u''(x) = -\frac{b}{R^2} e^{-x/R}$$

$$-\frac{u'(x)}{u''(x)} = R$$

Since

$$\rho(x) = -\frac{u'(x)}{u''(x)} = \rho_0 + Kx = R$$

we find $\rho_0 = R$ and $K = 0$.

b. $u(x) = a - \frac{b}{(1 + cx/R)^{1-c}}$

Resurrecting the power and quotient rules for derivatives from the deepest, darkest recesses of our memories, we find the following results.

$$u'(x) = b \frac{c}{R} \frac{1}{\left(1 + \frac{cx}{R}\right)^{1-c}}$$

$$u''(x) = -b \frac{c}{R} \frac{1-c}{\left(1 + \frac{cx}{R}\right)^{1-c}}$$

$$-\frac{u'(x)}{u''(x)} = R + cx$$

Accordingly, $\rho_0 = R$ and $K = c$.

c. $u(x) = a + b \ln(1 + x/R)$

$$u'(x) = \frac{b}{R} \frac{1}{1 + \frac{x}{R}}$$
Accordingly, \( r_0 = R \) and \( K = 1 \).

d. \( u(x) = a + bx \) \( \text{(expected-value decision-maker)} \)

\[
\begin{align*}
\frac{u'(x)}{u''(x)} &= R + x \\
-\frac{u'(x)}{u''(x)} &= R + x
\end{align*}
\]

The easiest way to find \( r_0 \) and \( K \) for this is to start with the exponential utility function, expressing the utility as a Taylor series, and then to take the limit as the risk tolerance \( R \) goes to infinity (the expected-value case). Accordingly, for an exponential utility function,

\[
\begin{align*}
u(x) &= a - \beta e^{-x/R} \\
u(x) &= \alpha - \beta + \beta \frac{x}{R} - \frac{\beta}{2} \left( \frac{x}{R} \right)^2 + \ldots
\end{align*}
\]

Setting \( b = bR \) and \( a = a + bR \), we obtain

\[
u(x) = a + bx - \frac{b}{2} x^2 + \ldots
\]

As \( R \to \infty \), all the terms except the first two go to zero, which means that \( K = 0 \) and \( r_0 = \infty \), yielding the infinite risk tolerance for the expected-value case.

The utility functions in b and c go to negative infinity when \( x = -r_0 / K \). What does this mean behaviorally? Is it meaningful to use these utility functions for values \( x \leq -r_0 / K \) ?

Behaviorally, \( x = -r_0 / K \) means ruin for that person, an outcome considered infinitely bad. Accordingly, the utility function produces nonsensical (and computer-confounding) results for values of \( x \leq -r_0 / K \) and should not be used in those cases.

5.17 If someone owns a venture, then his or her selling price for that venture should be such that he or she is indifferent between having the venture and having the selling price. The buying price should be such that the person is equally happy
(has equal utilities) before and after giving up the buying price and getting the venture.

a. Show that for a particular person the selling price of a venture is, by definition, equal to the certain equivalent of that venture.

By definition, the expected utility of a venture is

\[ E(u) = \sum_{i=1}^{n} p_i u(w + x_i) \]

where \( x_i \) represents each potential outcome and \( p_i \) is its corresponding probability. Given a current state of wealth \( w \) and a selling price (certain equivalent) \( S \), then indifference between having the venture and having the selling price (equal utilities) means that

\[ u(w + S) = \sum_{i=1}^{n} p_i u(w + x_i) \]

b. Show that if a person agrees to the Delta Property (which implies an exponential or linear utility function), then the selling price and buying price for a venture are the same for that person. (See problems 5.4 and 5.14 for a discussion of the Delta Property.) Does this result make sense to you?

The buying price for a venture must be such that the person is indifferent between his current state of wealth and his state of wealth with the venture and without the buying price, \( B \). Accordingly,

\[ u(w) = \sum_{i=1}^{n} p_i u(w + x_i - B) \]

\[ u(w + B) = \sum_{i=1}^{n} p_i u(w + x_i - B + B) \]

\[ u(w + B) = \sum_{i=1}^{n} p_i u(w + x_i) \]

\[ u(w + B) = u(w + S) \]

\[ B = S \]

By the Delta Property, the buying price can be added to all outcomes without affecting preferences.

5.18 A decision-maker is confronted with an uncertain venture \( A \) with outcome \( x_i \) and associated probabilities \( p(x_i | S) \).

a. Justify the statement that if the decision-maker already owns \( A \), then the minimum selling price \( S \) of the venture is equal to the certain equivalent.
ANSWERS TO PROBLEMS AND DISCUSSION NOTES

From problem 5.17a,

$$u(w+S) = \sum_{i=1}^{n} p_i u(w+x_i)$$

b. Justify the statement that the maximum buying price $B$ of the venture is a number such that if the decision-maker wishes to acquire $A$, his certain equivalent for $A$ with outcomes set to $x - B$ is zero.

From problem 5.17b,

$$u(w) = \sum_{i=1}^{n} p_i u(w+x_i - B)$$

$$0 = \sum_{i=1}^{n} p_i u(w+x_i - B) - u(w)$$

c. Show that $S = B$ for a linear utility function $u(x) = ax + bx$.

From 5.18a,

$$u(w+S) = \sum_{i=1}^{n} p_i u(w+x_i)$$

Using the function on both sides,

$$a + bw + bS = \sum_{i=1}^{n} p_i (a + bw + bx_i)$$

$$a + bw + bS = a + bw + b \sum_{i=1}^{n} p_i x_i$$

$$S = \sum_{i=1}^{n} p_i x_i$$

From 5.18b,

$$u(w) = \sum_{i=1}^{n} p_i u(w+x_i - B)$$

Using the utility function on both sides,

$$a + bw = \sum_{i=1}^{n} p_i u(a + bw + bx_i - bB)$$

$$a + bw = a + bw + b \sum_{i=1}^{n} p_i x_i$$
d. Show that $S = B$ for an exponential utility function $u(x) = a - b \ e^{-x/R}$.

For the selling price,

$$u(w + S) = \sum_{i=1}^{n} p_i u(w + x_i)$$

Using the utility function on both sides,

$$a - b e^{-(w+S)/R} = \sum_{i=1}^{n} p_i (a - b e^{-(w+x_i)/R})$$

For the buying price,

$$u(w) = \sum_{i=1}^{n} p_i u(w + x_i - B)$$

Taking the utility of both sides,

$$a - b e^{-w/R} = \sum_{i=1}^{n} p_i (a - b e^{-(w+x_i-B)/R})$$
ANSWERS TO PROBLEMS AND DISCUSSION NOTES

\[ e^{-B/R} = \sum_{i=1}^{n} p_i e^{-x_i/R} = e^{-S/R} \]

\[ B = S \]

e. Explain why if the stakes are large enough, the decision-maker can logically have \( S > B \).

If the stakes are very large relative to the decision-maker’s assets, then buying the venture would make the decision-maker poorer, thus making the certain equivalent very small or negative and yielding a selling price higher than the buying price.

5.19 Value of information can be viewed as the largest amount the decision-maker would be willing to pay to get the information. It is, therefore, really the buying price of the information. Show that the value of information is given by

\[ \text{Value of Information} = \text{Value with Information} - \text{Value Without Information} \]

for a straight line or exponential utility function. Explain behaviorally why this is not generally the case.

Without information, we have a venture with outcomes \( x_i \) and probabilities \( p_i \). With information, we have the same outcomes \( x_i \) but different probabilities \( p_i \) because we make decisions differently. Finally, we have the “Buy Information” alternative with outcomes \( x_i - B \) (where \( B \) is the price of the information) and probabilities \( p_i \). The maximum acceptable buying price for the information occurs when the following certain equivalents (CE) are equal.

\[ CE(\text{Buy Information}) = CE(\text{No Information}) \]

Linear utility function: Use the results of problem 5.21a to find the following results.

\[ CE(\text{Buy Information}) = \sum_{i=1}^{n} p_i' (x_i - B) \]

\[ CE(\text{Buy Information}) = -B + \sum_{i=1}^{n} p_i' x_i \]

Thus,

\[ B = CE(\text{With Information}) - CE(\text{Buy Information}) \]

Exponential utility function: Use the results of problem 5.21b to find the following results.

\[ CE(\text{Buy Information}) = -R \ln \sum_{i=1}^{n} p_i' e^{-(x_i - B)/R} \]
The result then follows just as with the linear utility function. It should be noted that we are seeing the Delta Property here. (See problems 5.4, 5.14, and 5.17.) The “Buy Information” alternative is just the “With Information” alternative, but with all outcomes shifted from \( x_i \) to \( x_i - B \). Hence,

\[
CE(\text{Buy Information}) = CE(\text{With Information}) - B
\]

If the cost of the information is too great, the decision-maker becomes so poor that he is out of the range in which the Delta Property applies—the “riskiness” or uncertainty appears more significant as the worst outcome combined with the selling price becomes more threatening.

5.20 Assume that you own two uncertain ventures A and B with outcomes \( x_i \) and \( y_j \), respectively. There are no synergies between the ventures, so the net outcome to the company is the sum of the outcomes for each venture. The probability distributions for A and B are independent.

\[ p(x_i, y_j \mid S) = p(x_i \mid S)p(y_j \mid S) \]

a. For a straight-line utility function \( u(x) = ax + bx \), show that the certain equivalent of A and B together is the sum of the certain equivalents for A and B separately.

The utility of two independent ventures A and B with outcomes \( x_i \) and \( y_j \), respectively, is given by:

\[
u = \sum_{i=1}^{n} \sum_{j=1}^{m} p_{ij}u(x_i + y_j) \]

\[
u = \sum_{i=1}^{n} \sum_{j=1}^{m} p_{ij}(a + bx_i + by_j) \]

\[
u = a + b \sum_{i=1}^{n} p_i x_i + b \sum_{j=1}^{m} p_j y_j \]
where \( p_i \) and \( p_j \) are the marginal probabilities obtained from \( p_{ij} \). The utility of the certain equivalent is given by

\[
u = a + bCE
\]

\[
CE = \frac{u - a}{b}
\]

Taking the formulation of the utility from above,

\[
a + b \sum_{i=1}^{n} p_i x_i + b \sum_{j=1}^{m} p_j y_j - a
\]

\[
CE = \frac{a + b \sum_{i=1}^{n} p_i x_i + b \sum_{j=1}^{m} p_j y_j}{b}
\]

\[
CE = \sum_{i=1}^{n} p_i x_i + \sum_{j=1}^{m} p_j y_j
\]

By the results of problem 5.21a, the right-hand side of this equation is the sum of the certain equivalents (expected values in this case) of A and B, respectively.

b. Repeat a for an exponential utility function \( u(x) = a - be^{-x/R} \).

For an exponential utility function,

\[
u = \sum_{i=1}^{n} \sum_{j=1}^{m} p_{ij} u(x_i + y_j)
\]

\[
u = \sum_{i=1}^{n} \sum_{j=1}^{m} p_{ij} \left( a - be^{-(x_i + y_j)/R} \right)
\]

\[
u = a - b \left( \sum_{i=1}^{n} p_i e^{-x_i/R} \right) \left( \sum_{j=1}^{m} p_j e^{-y_j/R} \right)
\]

where \( p_i \) and \( p_j \) are the marginal probabilities obtained from \( p_{ij} \). In this case, the utility of the certain equivalent is given by

\[
u = a - be^{-CE/R}
\]

\[
CE = -R \ln \left( \frac{a - u}{b} \right)
\]

Substituting in the formulation of \( u \) from above,

\[
CE = -R \ln \left[ \sum_{i=1}^{n} p_i e^{-x_i/R} \right] \left( \sum_{j=1}^{m} p_j e^{-y_j/R} \right)
\]
By the results of problem 5.21b, the right-hand side of this equation is the sum of the certain equivalents of A and B separately.

c. Explain in behavioral terms why the certain equivalent for A and B together can logically be less than the certain equivalents for A and B separately (e.g., for cases when a or b above do not apply).

It is the property in a and b above that lets the decision facilitator work on a decision problem without too much concern for the resolution of other decision problems within the company.

The cases illustrated in 5.20a and b would not apply when the individual worst outcomes are acceptable, but the combined results are not acceptable—such as buying a large number of independent futures contracts that could either make you a mint or leave you broke.

**5.21** You are given an uncertain venture with outcomes $x_i$ ($i = 1, 2, \ldots, n$) and probabilities $p(x_i|S)$.

a. Show that if the utility function is straight line $u(x) = a + bx$, then the certain equivalent of A is equal to its expected value.

For a straight line utility function,

$$u = \sum_{i=1}^{n} p_i(a + bx_i)$$

$$u = a + b\sum_{i=1}^{n} p_i x_i$$

But this utility is related to the certain equivalent by

$$u = a + bCE$$

$$CE = \frac{u - a}{b}$$

Therefore,

$$CE = \frac{a + b\sum_{i=1}^{n} p_i x_i - a}{b}$$

$$CE = \sum_{i=1}^{n} p_i x_i$$
b. If the utility function is exponential \( u(x) = a - be^{-x/R} \), show that the certain equivalent of \( A \) is

\[
CE(A) = -R \ln \left[ \sum_{i=1}^{n} p(x_i) e^{-x_i/R} \right]
\]

and is independent of the choice of values for \( a \) and \( b \) (provided that \( b \) is not equal to 0).

For an exponential utility function,

\[
u = \sum_{i=1}^{n} p_i u(x_i)
\]

\[
u = \sum_{i=1}^{n} p_i (a - be^{-x_i/R})
\]

\[
u = a - b \sum_{i=1}^{n} p_i e^{-x_i/R}
\]

This utility is related to the certain equivalent by

\[
u = a - be^{-CE/R}
\]

\[
CE = -R \ln \frac{a - \nu}{b}
\]

\[
CE = -R \ln \left( a - a + b \sum_{i=1}^{n} p_i e^{-x_i/R} \right) \frac{b}{b}
\]

\[
CE = -R \ln \sum_{i=1}^{n} p_i e^{-x_i/R}
\]

Because \( a \) and \( b \) cancel out, the choice of \( a \) and \( b \) does not matter except that because it is used as a divisor, \( b \) cannot equal zero.

5.22 Given the results of the previous problem, prove that for an exponential utility function there is the following approximate expression for the certain equivalent.

\[
\text{Certain Equivalent} = \text{Expected Value} - \frac{1}{2} \frac{\text{Variance}}{\text{Risk Tolerance}}
\]

Hint: Use the following Taylor Series approximations, which are valid for “small” \( x \).
\[ e^x = 1 + x + x^2 / 2! + x^3 / 3! + \ldots \]
\[ \ln(1 + x) = x - x^2 / 2 + x^3 / 3 - \ldots \]
In solving this problem, you only need to use the approximations as far as the squared terms. From problem 5.21b,
\[
CE = -R \ln \sum_{i=1}^{n} p_i e^{-x_i/R}
\]
\[
CE = -R \ln \sum_{i=1}^{n} p_i \left( 1 - \frac{x_i}{R} + \frac{x_i^2}{2R^2} + \ldots \right)
\]
\[
CE = -R \ln \left( 1 - \sum_{i=1}^{n} p_i \left( \frac{x_i}{R} - \frac{x_i^2}{2R^2} + \ldots \right) \right)
\]
\[
CE = -R \left\{ - \sum_{i=1}^{n} p_i x_i \frac{R}{R} + \sum_{i=1}^{n} p_i x_i^2 \frac{1}{2R^2} - \frac{1}{2} \left( \sum_{i=1}^{n} p_i x_i \right)^2 \right\} + \ldots 
\]
\[
CE = \sum_{i=1}^{n} p_i x_i \left( - \frac{1}{2} \sum_{i=1}^{n} p_i x_i^2 - \left( \sum_{i=1}^{n} p_i x_i \right)^2 \right) + \ldots 
\]
\[
CE = \text{Expected Value} - \frac{1}{2R} \text{Variance} + \ldots 
\]
5.23 You have the venture shown below and your certain equivalent is zero.

\[
\begin{align*}
& X \\
\left. \begin{array}{c}
\quad \rho \\
\downarrow \\
\quad \downarrow \quad 1 - \rho \\
\quad -X \\
\end{array} \right) \\
\end{align*}
\]
a. If your utility function is \( u(x) = a - bx^2 \), show that \( w^* = p/(1 - p) \). The value of \( p/(1 - p) \) is often called the odds.

The solution comes from the definition of certain equivalent.
\[
u(0) = pu(x) + (1 - p)u(-x)
\]
\[
a - bw^0 = p(a - bw^*) + (1 - p)(a - bw^*)
\]
The second solution is the one that applies.

b. If your utility function is \( u(x) = a - e^{-x/R} \), show that your risk tolerance is \( R = x / \ln (p/(1 - p)) \).

Substituting \( e^{-x/R} \) for \( w^{-x} \) in the equations for part 5.23a, we find

\[
e^{x/R} = \frac{p}{1 - p}
\]

\[
x/R = \ln \frac{p}{1 - p}
\]

\[
R = \frac{x}{\ln \frac{p}{1 - p}}
\]
A friend has come to you for help in deciding how to maximize his grade point average. What steps would you go through in developing a basis for the decision? What would you have at the end of the basis development? What are the next steps?

The important thing for students to realize is that the decision analysis cycle leads one through a process of narrowing possibilities and deepening detail. The process starts off with all possibilities wide open—the decision problem has yet to be defined. A basic definition of the problem is followed by an elicitation of possible options, which may or may not be complex enough to require a strategy table. In this example, possible decisions include which classes to focus on, whether to engage tutors, whether to take remedial courses, how to reflect potential grades in class selection, and the like.

Besides a listing of possible options, a student might structure a simple influence diagram to relate the various uncertainties and decisions together in a way that illustrates the value at stake (GPA). For instance, if homework is graded, does it affect the final grade directly, or only through the effect of doing or not doing homework on test scores?

The process should also produce at least a sketch of the value model. Even in the simple case of maximizing grades, there are trade-offs. Are you better studying late the night before a test, or would a full night’s sleep be of more benefit? Is attending discussion sections a better or worse use of time than just studying? The value model “sketch” should make a skeletal attempt to relate these various factors together.

The next step in the process would be to actually begin relating these factors together in an explicit value model (deterministic structuring). The model should use the described relationships to calculate a numerical value measure: the CPA. This allows explicit comparison of the relative importance
of variations in the different inputs as analysis proceeds into deterministic sensitivity analysis.

6.2 In the decision analysis cycle, there is a deterministic structuring phase. This phase often includes deterministic sensitivity analysis. In Chapter 3, there is a discussion of probabilistic sensitivity analysis. What are the differences between the two kinds of sensitivity analysis? What effects (or functions) does deterministic sensitivity analysis have in dealing with complex problems? Compare these two sensitivities with the sensitivity to risk tolerance seen in Chapter 5.

Deterministic sensitivity analysis is used to separate the wheat from the chaff, to identify those few crucial uncertainties that have the greatest impact on the final value measure. This identification of what is really important is often one of the most important products of the analysis, and it is crucial for structuring a manageable and meaningful tree. (Deterministic sensitivity analysis is also invaluable for examining the model to see if it produces reasonable results in a variety of circumstances.)

In contrast, probabilistic sensitivity analysis and sensitivity to risk tolerances perform a different function. They add a new dimension to the analysis, enabling examination of how the preferred decision varies with changes in the relative likelihood of possible outcomes (probabilistic sensitivity analysis) and with changes in the amount of risk that the person or company is willing to undertake (sensitivity to risk tolerance). Their primary use, then, is in probing the decision basis, rather than refining focus.

6.3 The complexity of a real-world problem is also reflected in its dynamic nature. The process of analyzing a decision problem can create new decision problems and add to the complexity of the original problem. A typical decision faced by the decision-maker (company) after the preliminary or pilot analysis is whether to proceed with the recommendation from the analysis or gather further information. Fortunately, the decision analysis cycle provides a framework with which to make this decision. Frequently, there are even preliminary numerical results available, such as the values of information.

What other complexities can arise in the course of analyzing the decision problem? (Hint: consider the elements of the decision basis.)

The many complexities that can pop up in the middle of the process have given rise to “war stories” of most professional decision analysts (“We cried about it through days of frantic work before we could laugh about it”). New, better alternatives often appear in the latter stages of the analysis and are one of the more welcome additions (though evaluating them may severely test the flexibility and credibility of the deterministic model). Probability distributions may change with the emergence of new information, or a different person may be designated as the expert for a particular variable.

The difficulties posed by these developments, however, pale beside those raised by value problems. A time value of money (discount rate) is a common
but Thorny problem. What do you do when a senior manager refuses to accept the separation of risk aversion from time value of money (the utility function vs. risk-adjusted discount rate battle)? Hidden values may emerge only in the very end to derail entire analyses. One recommendation to a major company was thrown out because, unknown to the project team, the board of directors refused to consider major redistributions of personnel as an acceptable choice. The computerization of logic becomes all the more crucial for coping with fast changes in alternatives, information, or values.

6.4 The ways of making decisions can be divided into normative methods and descriptive methods. Normative methods describe what people should do in a given situation. Descriptive methods focus on what people typically do.

For instance, if you face a decision on whether to hold on to a stock or sell it, a decision analysis (normative method) would tell you what you should do. Descriptively, many people make this kind of decision by asking their spouse, broker, or friends effectively to make the decision for them.

Is it possible to reconcile the two methods of decision-making? Provide an argument and example to support your judgment.

Normative decision making methods are simple and sensible enough that, when explained, most people will agree that they are desirable for appropriate decision problems (most people would not want to use a decision tree to choose a spouse, for example). The divergences from selections that normative methods would prescribe usually occur in situations where complexity and/or uncertainty make it difficult to see what the normatively better option would be—hence the importance of structuring, computers, trees, etc., to straighten things out. We would draw the conclusion, then, that if people understood the choices they were making, they would choose in accordance with normative guidelines, and the results from descriptive methods would then match those from normative methods. Although many would strongly disagree with this conclusion, criticism might be assuaged with the further observation that many problems are almost entirely questions of competing values—in which case the only “correct” answer is to pick the alternative that gives you the best combination of what you really want—if you can figure out what you really want.

6.5 The framing of a decision problem describes how the decision is stated. An example is describing the effects of a new, dangerous, and relatively untested drug either in terms of net lives saved or net lives lost. Decision-makers are often affected by the framing of the problem. For example, the information they provide and preferences they express will vary with the framing of the problem. Many of these framing issues are believed to be psychological in origin. What could you do to avoid these problems?

Like many psychological biases, problems that arise from framing can be ameliorated by considering the problem from different angles. One good
method for coping with this problem is thinking of all the different groups that could be affected by a given decision and then considering how they would view the problem. In the case of a new drug, one could consider how the patients, patients' families, doctors, insurance companies, employers, and researchers would be affected. While an entirely neutral problem formulation can probably never be achieved, formulations are possible (and desirable) that aid meaningful analysis without unduly prejudicing the results. A formulation that carefully considers the relevant factors, perhaps from several viewpoints (without losing sight of the decision-maker's values) is probably the best preventative against problems arising from framing.

6.6 Innovative Foods Corporation (IFC) is a wholly owned subsidiary of Universal Foods Corporation (UFC). UFC is a major Fortune 500 company in the food processing industry. IFC is in the market of supplying specialized processed foods for human consumption. In 1979, the total market for specialized processed foods amounted to $600 million and is growing.

IFC’s leading products are dehydrated and processed foods targeted at two consumer groups: people on a special diet and people who are recovering after a serious illness. IFC has been using a well known additive in its food processing, Divit, which is a recognized food additive in the food processing industry.

By carefully researching and testing its products, IFC has established a secure but small market share. IFC has a reputation as a good and reliable company and anticipates a $2 million net yearly cash flow after taxes for the next 20 years.

In the last year, IFC management has become acquainted with some troublesome experiments carried out by its research division. The results of these experiments indicate a high probability that Divit is carcinogenic when applied in very high concentrations to the skin of mice. The carcinogenic properties when ingested by humans are by no means certain. IFC believes its competitors may be on the same track. If Divit turns out to be carcinogenic, the FDA will surely ban it, thus bringing about a major decrease in IFC’s earnings and probably the loss of most of IFC’s hard-earned market share.

However, the IFC management has also been informed about another option. Its research division has developed another additive, Biovit. The research director believes Biovit is an excellent food additive that will have none of the problems of Divit. At present, the manufacturing costs of Biovit are uncertain. To process its food with Biovit, IFC will have to invest around $150 million. IFC management must carry out an important decision: should it continue to use Divit and face its potential banning or should it invest in new facilities and start using Biovit?

Assume you are a member of IFC’s executive committee. How would you structure your thinking about this problem? Are there ethical considerations?
One possible way of structuring the problem is shown below. In this diagram, we see that IFC profits are affected by Divit profits, by future lawsuits, and by Biovit investment and profits. These factors are, in turn, affected by other uncertainties and decisions. The problem could be structured in a number of different ways, depending on the views of the person doing the structuring. One might, for instance want to add the effect of an FDA ban on other IFC product lines, or add the macroeconomic variables of market size or market share, or express the consequences of Divit being carcinogenic in terms other than the cost of future lawsuits.

The important point is that continuing to market Divit is a decision, just as discontinuing it or pursuing more testing is a decision, and all these decisions have potential positive or negative consequences. The costs of additional testing or development, future sales and profits, and the effects of people taking Divit or Biovit all affect the net financial consequences to IFC.

This last point raises an ethical question regarding the continuing sale of Divit. Is it ethically permissible to even consider continuing to sell Divit when there is evidence of it causing cancer? Some might think it equally objectionable to reduce the effect of Divit on people to the monetary cost of future lawsuits. One way to approach this problem is to focus on the perspectives of the parties involved.

From IFC’s perspective, the authors would argue that there is nothing
wrong with including the cost of future lawsuits as a cost of continuing to market Divit. The managers of IFC have a responsibility to the shareholders to maintain or improve the value of their investment, primarily through the company’s profits. Therefore, they certainly should consider all the factors that could significantly affect profitability, including lawsuits and their effect on the company’s reputation. Financial consequences are relevant in analyzing the decision and are a partial measure of society’s judgment of the trade-off.

What other costs and considerations must be included? Ethical considerations must also be put in the value function (though the decision-maker may not wish to put them on paper). Almost all actions/products give some danger to others, and, for society to function, some trade-offs must be made. The problem of determining what level of trade-off is acceptable is an important one that goes far beyond the scope of this book.

6.7 Plastic Co. is a fairly large company that manufactures bulk plastics for a large variety of uses. It has an extensive network of customers—companies that turn the bulk plastic into items that are then sold to the end-user.

Tech Co. is a European company that owns a process that can make a special plastic that is useful for making bearings for high-speed centrifuges. There are several other potential high-tech applications for this special plastic. In addition, the same production equipment can also be used for making a common, low-margin type of plastic, which, it turns out, is not a plastic that Plastic Co. currently makes.

Tech Co. does not want to enter the U.S. market and is offering Plastic Co. an exclusive license for the manufacturing technology. The asking price is a $500,000 license fee plus 5 percent of sales for 10 years.

Plastic Co. has determined that the equipment could come in a small size (3 million pounds per year) or a large size (10 million pounds per year). The respective costs for the equipment are $3 million and $7 million. Tech Co.’s experience has been that production costs are $0.20 per pound.

a. Begin to develop the decision basis for this decision. What are the alternatives? What are the uncertainties? What is the value? What information do you have? What information do you still need?

As with other questions in this chapter, a number of answers are possible. The object is for the student to start with the initial information and structure it with necessary additional information to form a complete pilot analysis. One way of representing the problem is with a decision on whether to buy the small equipment, large equipment, or no equipment. The uncertainties could include the potential sales of both the special and common plastics, and the final profit margins on each (you would probably sell as much of the high-margin special plastic as possible and fill in the rest of the production capacity with common plastic). The value could be to maximize the expected net present value of the investment, given that the investment could also be made.
in something other than new manufacturing equipment. The information given includes investment and production costs and capacities, and licensing fees. Information is still needed on non-production costs, on market size and share for each product, and on possible returns from a suitable alternative investment.

Distribution is also important in this type of problem. An important consideration is how to establish distribution of the new product, while not disturbing the distribution of current products.

b. Designate someone to be the decision-maker. She will be the president of Plastic Co. Review with her the work you have done so far and finish structuring the decision. Assess any further information you may need. Answers to this question should fill in any holes in the decision basis as identified in 6.7a. The result should be a completely filled-in prototype decision tree, suitable for analysis. One such influence diagram and tree (without additional numbers filled in) is shown below.
c. Analyze the decision and produce a recommended course of action. Review the recommendation with the decision-maker. (Does she need to know the details of the analysis?) Is your recommendation useful to her? Does she believe and understand it? Why or why not?

Answers to this part might consist of a summary version of a presentation and should focus on qualitative conclusions and reasoning brought out by the quantitative analysis ("On the whole, the prospects for large-scale marketing of the special plastic are most promising because..."). The decision-maker usually neither needs nor desires to know the details of the analysis—sometimes not even the results of sensitivity analysis or the shapes of probability distributions. The usefulness and believability of the recommendation will hinge on how well it addresses the concerns she raised and on whether it accurately incorporates her and others’ information to reach logical, sensible, and possibly surprising conclusions. A relevant, cogent, and authoritative recommendation has the best chance of being acted on.

6.8 Form a group to analyze a decision about whether or not to add a salad bar to a pizza parlor. Designate at least one person to be the client and one person to be the analyst. (You may have more than one of each.) Make a pass through the decision analysis cycle as described below.

a. **Background.** Develop an image of the pizza parlor that is as realistic as possible for whoever is playing the client. (This will greatly aid in the assessments.) How large is the place? How old is it? Who owns it? Who runs it? What kind of an area is it located in? What kind of clientele does it have? What is currently on the menu? What is the monthly sales volume? How many customers does it have daily? What are the peak hours? What kind of decor does it have?
One good way to organize answers to this question is in the form of a five- to ten-page presentation or report. The presentation could be in a form suitable for overhead projectors (brief titles and explanatory points), or could be in text form. This slide, for instance, could focus on the economics of this pizza parlor, mentioning what the market is, how it works, and what is necessary to succeed in this market (as much as can be done in a hypothetical case).

b. **Basis Development.** Develop the basis for the decision. If you like, you may use an influence diagram for this step. What decisions must be made? What are the significant uncertainties? How do they relate to one another? What are the values on which the decision will be made? Try to keep the problem description simple.

This slide could lay out the basis of the decision, perhaps in the form of an influence diagram. A preliminary tree drawing may be useful at this stage, but would often be complicated and confusing because sensitivity analysis has not yet been used to prune it. Whatever method or combination of methods is used, it should specifically answer each question posed in 6.8b.

c. **Deterministic Structuring.** Develop a model to determine the value for any scenario the tree might generate. The model may be assessed values, a Basic endpoint expression in Supertree, or an external spreadsheet model. Use sensitivity analysis if necessary to reduce the number of variables in the tree. Focus on modeling to help your understanding of the problem and to distinguish between alternatives.

These pages may contain a printout of part of a spreadsheet, but a block diagram of the main modules of the model is often easier to understand. The model should be capable of realistically showing the effects of variation in all the inputs. Deterministic sensitivity analysis results should be presented to determine which uncertainties are critical—a computer printout would be fine. A nice touch here would be the addition of some joint sensitivities if there are strong interactions between any of the variables (“What happens to profits if we add a salad bar and get a wine and beer license?”).

d. **Probabilistic Evaluation.** Build and analyze the decision tree. For simplicity, try to keep the number of nodes down to four or five. You may start with a larger tree and then eliminate the nodes that do not distinguish between alternatives. Examine profit distributions, expected values, tree drawings, probability sensitivities, etc. Check that the results are consistent with your understanding of the problem.

The analysis results presented should not be an exhaustive exercise of every feature in Supertree. Rather, the results should be limited to those that lend the most insight and that will form the basis for the conclusions drawn in the next step.
e. **Basis Appraisal.** What is the preferred decision? How does its expected value and risk compare to the other alternatives? Is the preferred decision sensitive to changes in probabilities or risk attitude? What are the values of information and control? Would the client feel comfortable acting at this point, or would further study be advisable?

The slide(s) here should present the conclusions of the analysis and critically examine where and why the preferred alternative is better or worse than the other alternatives. This section should conclude with either a statement that the problem seems well-enough understood to act upon the recommendation or that critical new questions have been raised that require further analysis.

f. **Action.** Prepare a list of requirements for implementing the recommended alternative. These may include allocating funding, hiring personnel, hiring contractors, etc., or there may be no requirements if the recommendation was to do nothing. Has the analysis shed any light on the steps required for implementation? Is there any value to updating the analysis periodically to provide further guidance?

If the alternatives were carefully thought out, then selection of one should immediately suggest the steps to implement it. Similarly, the possibilities examined in other alternatives that produced little expected value or return should be discarded. The analysis may also have suggested possible scenarios that would require reexamination of the preferred alternative ("If the salad bar fails to bring in at least an additional $25,000 in business per month, then strategy 3 should be reevaluated.")

6.9 Form a small group to perform a decision analysis of a case study you have previously worked on. Assign roles. You will need at least one analyst and one client who can supply structure and probabilities. Complete at least one pass through the cycle, perhaps limiting the exercise to two or three hours. Spend most of the time structuring the problem and preparing a final report.

Refer to the suggestions in the previous question.

6.10 Insitu Corp., an energy company, had developed a new technology for oil drilling in cold climates. The technology involved injecting a heated chemical solution into the well field at one location and waiting for the solution to percolate through the oil-bearing formation. Then the solution was pumped out of the well field at another location and the oil was extracted in a processing plant.

Insitu had proven this technology on a pilot scale and was considering whether to build a full-scale project on its Whalebone property in Alaska. One of the major uncertainties was the capital cost of constructing the complex consisting of the plant, pipeline, and well field. An engineering and design firm had estimated a base cost of $320 million. To obtain financial backing for the project, Insitu felt it needed to verify this cost.
A team assembled in September to review the cost estimate and identified two major risks in the estimate. First was the question of the efficiency of the new technology. In the pilot plant, a flow rate of 500 gallons per minute had produced a solution 30 percent saturated with oil. However, if the full-scale process were less efficient and produced, say, only 10 percent saturation at a flow rate of 1,000 gallons per minute, additional equipment would be required for the volume of oil produced to remain constant. Engineering estimates the additional equipment would add 15 percent to the base cost.

The second major risk was the productivity of the union workers. The largest influence on productivity was the unemployment rate in the area. If unemployment were high, then the workers would be less likely to strike and would work harder. Unemployment, in turn, depended on the number of large pipeline and energy projects competing for workers and, ultimately, on energy prices. Changes in energy prices in recent years had been correlated with productivity variations as large as ±30 percent. The team decided to include in its estimate a contingency to reflect these risks in the capital cost.

As the team was about to adjourn, someone asked if there were any other reasons the base cost could be exceeded. An inexperienced staffer, Ms. Pessi Mist, asked whether they were sure the construction would be finished on time. Since wage and materials rates were escalating at almost 25 percent a year, she felt a late construction schedule would increase costs. Her question was met with disbelief. The venture manager explained that most of the construction had to be completed before the spring thaw date, because heavy equipment could not be operated on the muskeg once it thawed in June. The EPA was very unlikely to allow summer construction on the fragile muskeg. In
addition, a June 1 expected completion date had already been announced publicly by the President of Insitu. No one had to mention the company’s unblemished record of completing projects within the allotted time once construction was under way. Because of the cost of interest on funds expended during construction, Insitu had made this its trademark.

Undaunted, Ms. Mist pursued the question of what remained to be done before the three-month construction schedule could begin. A cost engineer explained that the board of directors had taken the position that it would not meet to review the project unless the native claims issues were settled for the pipeline route. Without board approval, a contract could not be let. If the contractor did not arrange for materials delivery to the site by March 1, the start of the project would be delayed. In addition, the board required a minimum of one month for deliberation, and two months each were required to let contracts or deliver materials.

Sparked by the mention of the EPA, another young staffer, Enviro Mann, asked what would happen if the EPA did not allow the spent solution to be pumped back into the mine shaft as planned. The environmental engineer assured him that a waste pond would cost only $5 million to build. The possibility that recycling of the solution would be required was very remote.

a. Draw an influence diagram for the total capital cost of the complex in current dollars.

Slightly unsettled by the questions of Ms. Mist and Mr. Mann, the team assigned them the job of developing a better picture of the risks in the capital cost. Mist
and Mann interviewed a number of people in the corporation. From the engineering manager who knew the most about scaling up chemical processes, they assessed a 75 percent chance that the full-scale plant would work as efficiently as the pilot plant. From the regulatory affairs department, they assessed a 50 percent chance that the spent solution could be put in the mine shaft. There was only a 20 percent chance that recycling would be required. However, if this additional step were required in the process, $120 million of equipment would be added. The manager of regulatory affairs was uncomfortable about whether the EPA would allow summer construction on the muskeg. He could remember only five winter Alaskan projects that had been delayed until summer. Of these, only one had been allowed to proceed before the September 1 freeze date. For the four events necessary to begin construction, the following probabilities were assessed.

<table>
<thead>
<tr>
<th>Event</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Native claims issues settled for pipeline route by October 1</td>
<td>.70</td>
</tr>
<tr>
<td>Board of Directors approval by November 1</td>
<td>.50</td>
</tr>
<tr>
<td>Contract let by January 1</td>
<td>.90</td>
</tr>
<tr>
<td>Materials on site by March 1</td>
<td>.25</td>
</tr>
</tbody>
</table>

The probability distribution below was assessed for worker productivity. Ms. Mist noted that labor costs were only 35 percent of the total construction costs.
b. Draw a probability tree for the total capital cost of the project in current dollars. Label the branches and put in the probabilities.

The cost multiplier data in the tree above were obtained in the following manner: The productivity multiplier was discretized from the graph above to obtain the values 1.5, 1.0, and 0.4 with probabilities of .25, .50, and .25, respectively. However, the productivity multiplier affects only the 35 percent of total construction costs represented by labor costs. Therefore, the values for the probability distribution for the cost multiplier are $(0.65 + 0.35 \cdot 1.5)$, $(0.65 + 0.35 \cdot 1.0)$, and $(0.65 + 0.35 \cdot 0.4)$ or 1.2, 1.0, and 0.8.

The definition of the product multiplier in the graph does not pass the clairvoyance test. It is intended to be a multiplier on cost ($/\text{unit of work}$). However, students could interpret it as a multiplier on productivity (work$/\text{unit}$); in this case, the cost multipliers would be $(0.65 + 0.35/1.5)$, 1, and $(0.65 + 0.35/0.4)$.

c. Write an equation for the cost model to calculate the total capital cost of the project.

\[
\begin{align*}
\text{Total Capital Cost} &= \left( \frac{\text{Base Cost}}{\text{Cost Factor}} \right) \times \left( \frac{\text{Process Efficiency}}{\text{Escalation Factor}} \right) + \left( \frac{\text{Disposition Cost}}{\text{Escalation Factor}} \right) \times \left( \frac{\text{Cost Multiplier}}{\text{Escalation Factor}} \right) \\
&= \left( \frac{\text{Base Cost}}{\text{Cost Factor}} \right) \times \left( \frac{\text{Process Efficiency}}{\text{Escalation Factor}} \right) + \left( \frac{\text{Disposition Cost}}{\text{Escalation Factor}} \right) \times \left( \frac{\text{Cost Multiplier}}{\text{Escalation Factor}} \right)
\end{align*}
\]

Note that cost escalation factor is 1.0 if the project finishes on time, 1.06 if the project is late but the EPA allows summer construction, and 1.12 if the project is late and the EPA does not allow summer construction.

d. Calculate a probability distribution on the total capital cost of the project in current dollars. How do you explain its shape?
Tree name: Insitu Corp.

<table>
<thead>
<tr>
<th>STRUCTURE</th>
<th>NAMES</th>
<th>OUTCOMES</th>
<th>PROBABILITIES</th>
</tr>
</thead>
<tbody>
<tr>
<td>1C2</td>
<td>Finish</td>
<td>1.06</td>
<td>.08 .92</td>
</tr>
<tr>
<td>2C3</td>
<td>EPA</td>
<td>Depends on 1</td>
<td>.2 .8</td>
</tr>
<tr>
<td>3C4</td>
<td>Efficiency</td>
<td>1.15</td>
<td>.75 .25</td>
</tr>
<tr>
<td>4C5</td>
<td>Disposition</td>
<td>120.0</td>
<td>.2 .8</td>
</tr>
<tr>
<td>5C6</td>
<td>Cost</td>
<td>1.2</td>
<td>.50 .25</td>
</tr>
<tr>
<td>6E</td>
<td>$((320<em>Efficiency)+Disposition)</em>(Finish+EPA)*Cost</td>
<td>Depends on 1 2 3 4 5</td>
<td></td>
</tr>
</tbody>
</table>

Finish OUTCOME, NODE 2
1 0 0
1.06 0.06

The expected value of this tree is $391 million, and the probability distribution is shown below.

Some of the lumpiness in this distribution is caused by the way the inherently continuous variables (such as the union productivity multiplier) have been discretized into chance nodes. However, whether construction is finished by June 1 and whether the EPA allows summer construction are inherently discrete variables and would always account for jumps in what might otherwise be a smooth curve.
The shape of the curve itself reflects the multiplicative nature of the value function. When the project is finished on time, there is no escalation factor increasing all of the other costs. This accounts for the low-probability region of low costs. When a late finish begins to multiplicatively compound the other costs factors, the product (total cost) increases rapidly, eventually trailing off into the high-cost, low-probability scenarios where, on top of everything else, the spent solution must be recycled. As will be seen below, costs increase dramatically when this happens.

\[ e. \quad \text{Perform a sensitivity analysis to determine the most important risks in the total cost. Calculate the following quantities.} \]

- The change in total expected cost when each individual variable changes from its lowest to its highest value. This answers the question: “How much difference does this variable make in the expected cost?”

Note that all of the following plots have been done with the same limits on the vertical axis so that they are directly comparable to one another.

For whether or not the project is finished by June 1, the potential change in expected cost is $394 - 356 = \$38\text{ million}.

For whether the EPA allows summer construction, the potential cost difference is $395 - 376 = \$19\text{ million}.
For process efficiency, the potential cost difference is $431 - 378 = $53 million.
For disposition of spent solution, the potential cost difference is $497 - 365 = $157 million.

For union productivity multiplier, the potential cost difference is $470 - 313 = $157 million.
• The expected change in the standard deviation of the total cost if perfect information were available on each variable. This answers the question: "How much does this variable contribute to the risk?" Why can’t we do ordinary value of information calculations?

These quantities can be obtained by using the List Distribution Values command to obtain the original variance, then setting the probabilities for each chance node to 1 and 0 one at a time and relisting the distribution to obtain the new variance. Alternatively, you could select the List Distribution Values command, change the order of nodes to bring the desired node to the front of the tree, and list the distribution at the second node (specifying the desired branch of the first node). Taking the square root of the variance will produce the standard deviation. Ordinary value of information calculations are not possible because there are no decisions in the tree.

<table>
<thead>
<tr>
<th>Tree</th>
<th>Variance</th>
<th>Standard Deviation</th>
<th>Difference from Original</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original</td>
<td>6,620</td>
<td>81.4</td>
<td>—</td>
</tr>
<tr>
<td>Finish to 0.1</td>
<td>6,614</td>
<td>81.3</td>
<td>0.1</td>
</tr>
<tr>
<td>EPA to 0.1</td>
<td>6,706</td>
<td>81.9</td>
<td>-0.5</td>
</tr>
<tr>
<td>Efficiency to 1.0</td>
<td>5,872</td>
<td>76.6</td>
<td>4.8</td>
</tr>
<tr>
<td>Disposition to 0.1</td>
<td>3,354</td>
<td>57.9</td>
<td>23.5</td>
</tr>
<tr>
<td>Cost to 0.1,0</td>
<td>3,486</td>
<td>59.0</td>
<td>22.4</td>
</tr>
</tbody>
</table>

f. What conclusions, insights, and recommendations would you make for risk management and cost control?

The relative differences made in the standard deviation give the relative importance of each uncertainty. The uncertainties on cost (worker productivity) and on disposition of the spent solution have the strongest effect by far on the risk. Students might also use the swings from the probability sensitivity graphs (above) to construct a chart showing the square of each swing as a percent of the sum of squares, as was done in Figure 6-10.

Recommendations to management, then, could include a suggestion of focusing attention on the problems of worker productivity and of disposition of spent solution. These problems dwarf other concerns.
Huntenpeck Company manufactures typewriters, and Huge company has requested a bid on a contract for 10,000 new typewriters, which cost $1,000 to manufacture. Huntenpeck would very much like the contract, especially since the publicity would lead to additional sales to Huge’s subsidiaries. On the other side of the coin, losing this large contract would introduce a competitor with the potential to cut into Hun ten peck’s sales of dicta phones to Huge Company. The secondary impact of winning or losing the contract would be felt for about five years.

a. Begin structuring the problem and decide what information you need. Make sure you draw an influence diagram.

The influence diagram below shows the structure of the decision problem. Information will be needed to describe each of the nodes. The decision node, Huntenpeck Bid, will need a description of the alternatives. The chance nodes will need information on the possible outcomes and the probabilities of occurrence. The NPV node will need information on costs, tax rate, discount rate, and the like.
Huntenpeck decided to make its decision on purely economic factors and to be an expected-value decision-maker.

After heated discussion, Huntenpeck estimated that if it won the contract, it would sell between 1,000 and 1,400 extra typewriters a year to Huge’s subsidiaries over the next five years, with equal probabilities of the high and low figures. These typewriters would be sold at $2,000 (in 1986 dollars). If it lost the contract, it would lose about $1 million profit a year in dicta phone sales over the next five years. After even more heated discussion, Huntenpeck decided to use a discount rate (time value of money) of 10 percent for constant dollar analysis.

Concerning the contract itself, there seemed to be two principal competitors: Carboncopy and Misprint. No one was sure what their bids would be. However, their typewriters were similar enough to Huntenpeck’s that it was certain that Huge would accept the lowest bid. Probability distributions were encoded for both competitors’ bids.

<table>
<thead>
<tr>
<th>Bid Level ($ per typewriter)</th>
<th>Probability of Competitor Bidding Lower</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Carboncopy</td>
</tr>
<tr>
<td>$1,200</td>
<td>.01</td>
</tr>
<tr>
<td>$1,300</td>
<td>.05</td>
</tr>
<tr>
<td>$1,400</td>
<td>.20</td>
</tr>
<tr>
<td>$1,500</td>
<td>.50</td>
</tr>
<tr>
<td>$1,600</td>
<td>.80</td>
</tr>
<tr>
<td>$1,700 (4.17)</td>
<td>1.00</td>
</tr>
</tbody>
</table>

b. Finish structuring the problem and draw the decision tree for it.

The tree below represents the problem.

A word is in order about how the tree above was produced and how it works. First of all, a little work is needed before the probabilities given in the problem can be used. Instead of having a chance node on Carboncopy’s bid and one on Misprint’s bid, you could have a chance node on whether or not Huntenpeck wins the contract. The probability of Huntenpeck winning for a given bid level is the product of the probabilities of Carboncopy and Misprint bidding higher, which is one minus the given probabilities of their bidding lower. Thus, for a bid of $1,200, Huntenpeck would have a probability of .99
1.0 = .99 of winning. Similarly, for a bid of $1,500, Huntenpeck would have a probability of winning of \( .50 \cdot .55 = .28 \), and a probability of .72 of losing.

However, the difficulty presented by this approach is that the chance node on winning would have probabilities dependent on the level of bid, which means that the order of nodes could not be reversed for value of information calculations. An alternate approach is to input separate, independent chance nodes for each competitors’ bids. This can be obtained by recognizing that the information given is in the form of a cumulative probability distribution, which can be plotted (they make nice, smooth curves) and discretized, as shown at the end of Chapter 2, to obtain the probabilities and outcomes shown above. The order of these nodes can then be changed to obtain the value of information on one or both competitors’ bids.

The outcomes for the extra business node are the present values of the two potential levels of extra business. Thus the present value at 10 percent of \( ($2,000 – $1,000) \cdot 1,000 \) typewriters is $4.17 million, while the present value of 1,400 typewriters per year is $5.84 million. The present value of dictaphone profits that would be lost if the contract is lost is $4.17 million.

The Basic-syntax endpoint expression first of all compares Bid to CarboncopyBid and MisprintBid. The logical expression \((A < B)\) is equal to 1 if the expression is true and zero if it is false. If Bid is greater than or equal to either one of those, then Huntenpeck has lost the contract and the $4.17 million present value of lost dictaphone business is the value of the expression. If Huntenpeck has won the contract, the $4.17 million is added in at the beginning of the expression and subtracted in the second term, yielding zero for the lost business.

The expression also calculates the profit if Huntenpeck has won. The profit is the extra typewriter business plus .01 million (10,000) typewriters times the profit margin of bid minus the $1,000 cost. The result of all this is a calculated profit for each scenario. Of course, the whole thing could have been done in a spreadsheet with IF statements (see the solution for problem 3.22).

c. What is Huntenpeck’s optimal bid ($ per typewriter)?
As shown below, the optimal bid is $1,300 per typewriter. The probability distribution for this option is shown by the 2’s in the plot. Note that bidding $1,600 and $1,700 have the same expected value because, in both cases, you are sure to lose the contract.

<table>
<thead>
<tr>
<th>Bid</th>
<th>Exp Val</th>
</tr>
</thead>
<tbody>
<tr>
<td>1200</td>
<td>6.95</td>
</tr>
<tr>
<td>&gt; 1300</td>
<td>7.52</td>
</tr>
<tr>
<td>1400</td>
<td>5.65</td>
</tr>
<tr>
<td>1500</td>
<td>-2.07</td>
</tr>
<tr>
<td>1600</td>
<td>-4.17</td>
</tr>
<tr>
<td>1700</td>
<td>-4.17</td>
</tr>
</tbody>
</table>

**Expected Value:** 7.52
The probability distribution is difficult to read because of the multiplicity of bids and the discontinuities in the outcomes. However, you can see that the bid of 1300 does not have the upside potential of the higher bids, but it also has less risk of losing the bid.

d. What are the values of information for the crucial uncertainties? Are there circumstances under which the optimal bid would be different given perfect information?
As shown below, if you knew Carboncopy’s bid, Huntenpeck’s optimal bid would be either $1,400 or $1,300. The value of information on Carboncopy’s bid is $7.91 - 7.52 = .39, or $39,000.
Knowing Misprint's bid does not change Huntenpeck's preferred bid. Accordingly, the expected value is the same and the value of information is zero.
Again, knowing the amount of extra business does not change the preferred decision and there is no value of information.

\[
\begin{array}{c|c|c|c}
\text{Bid} & \text{Exp Val} \\
\hline
1200 & 6.12 \\
1300 & 6.72 \\
1400 & 5.03 \\
1500 & -2.19 \\
1600 & -4.17 \\
1700 & -4.17 \\
\hline
\end{array}
\]

\[
\begin{array}{c|c|c|c}
\text{Bid} & \text{Cert Eq} \\
\hline
1200 & 6.87 \\
1300 & 7.07 \\
1400 & 3.70 \\
1500 & -2.98 \\
1600 & -4.17 \\
1700 & -4.17 \\
\hline
\end{array}
\]

\[
\begin{array}{c|c|c|c}
\text{Prob} & \text{Extra Exp Val} & \text{Bid} & \text{Exp Val} \\
\hline
0.500 & 4.17 & 6.72 \\
0.500 & 5.84 & 8.32 \\
\hline
\end{array}
\]

e. If Huntenpeck had a risk tolerance of $10 million, what would its optimal bid be?

As shown below, the optimal bid is still $1,300, and the certain equivalents have not changed very much from the expected values. A further exercise would be to conduct risk sensitivity analysis to see if the preferred decision would ever change.

\[
\begin{array}{c|c|c}
\text{Bid} & \text{Cert Eq} \\
\hline
1200 & 6.87 \\
1300 & 7.07 \\
1400 & 3.70 \\
1500 & -2.98 \\
1600 & -4.17 \\
1700 & -4.17 \\
\hline
\end{array}
\]

7.2 The ABC Construction Corporation is being sued for damages as a result of an accident in which the plaintiff fell from a second-floor balcony. The open side of the balcony had been closed off only by a pair of chains held in place by hooks at each end. The plaintiff was leaning against the chains when one of the hooks snapped. He incurred serious injuries in the subsequent fall. In his complaint, the plaintiff charges ABC Construction Corporation with negligence in designing the balcony and the Wisconsin Hook Company with negligence in manufacturing the hooks and asks for $2 million in damages, including pain and suffering.

ABC has a $3 million policy covering this kind of claim with United Insurance Company. United’s attorneys have told the claims supervisor that a jury might find ABC negligent in this case. Furthermore, they have said that independent of the jury’s finding about ABC’s negligence, the hook manufacturer could be found negligent. If both defendants are found negligent, each would have to pay 50 percent of the total damages awarded to the plaintiff. If only one is found negligent, that defendant would have to pay the full award. It is believed
that the jury would probably award only $500,000 or $1,000,000, but there is some chance of the full $2,000,000 award.

United Insurance Company has been approached by the plaintiff’s counsel and been given the opportunity to settle out of court for $500,000. Should the offer be accepted?

a. Structure the litigate/settle problem as an influence diagram and then draw the decision tree.

The influence diagram and tree below show one possible way of structuring the problem.

\[
\text{Strategy} \quad \text{Verdict} \quad \text{Share} \quad \text{Award} \\
\text{Litigate} \quad \text{Negligent} \quad 0.5 \quad \$500,000 \\
\text{Not Negligent} \quad 1.0 \quad \$1,000,000 \\
\text{Settle} \\
\]

b. Assign dollar outcomes to all end points of the tree.

See the tree drawing in 7.2c.

Noticing that litigation could produce losses substantially in excess of the $500,000 settlement offer, the claims supervisor realized that he had better quantify the likelihood of the various outcomes. After a long, rigorous discussion
of the legal and damage issues, the lawyers provided the claims supervisor with the following probabilities.

There is a 50 percent chance of a jury finding ABC negligent, but only a 30 percent chance of finding the hook manufacturer negligent. If both defendants are found negligent, there is a one chance in five of the full $2,000,000 being awarded. The $500,000 and $1,000,000 awards are equally likely. If only one defendant is found negligent, on the other hand, the probability of the full $2,000,000 award is cut in half, and there is a 60 percent chance of the $500,000 award and a 30 percent chance of the $1,000,000 award.

c. Find the expected value of each of United Insurance Company’s alternatives.

The tree as input into Supertree is shown below. Note that this tree has the costs input as positive numbers, and, accordingly, needs to have the decision criterion in Supertree set for minimization.

As illustrated below, the expected cost of litigating is $355,000, versus the sure $500,000 cost of settling now.

d. Draw the profit distributions for United Insurance Company’s alternatives.
The attorneys were surprised to hear that the claims supervisor was going to reject the settlement offer. Because they felt that a little more pretrial discovery could greatly reduce their uncertainty about whether or not the hook manufacturer would be found negligent, they urged the claims supervisor to briefly delay his decision until they could complete some additional pretrial work.

**e. Determine the expected value of perfect information about whether the hook manufacturer will be found negligent.**

As illustrated below, the preferred decision is to litigate regardless of whether the hook manufacturer is found negligent. Accordingly, the expected value is the same and there is no value of information.

<table>
<thead>
<tr>
<th>Probs Share</th>
<th>Exp Val</th>
<th>Strategy</th>
<th>Exp Val</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.300 5</td>
<td>250</td>
<td>Litigate</td>
<td>250</td>
</tr>
<tr>
<td>0.700 1</td>
<td>400</td>
<td>Litigate</td>
<td>400</td>
</tr>
</tbody>
</table>

**Expected Value: 355**

7.3 **Hony Pharmaceuticals is a manufacturer engaged in developing and marketing new drugs. The chief research chemist at Hony, Dr. Bing, has informed the president, Mr. Hony, that recent research results have indicated a possible breakthrough to a new drug with wide medical use. Dr. Bing urged an extensive research program to develop the new drug. He estimated that with expenditures**
of $100,000 the new drug could be developed at the end of a year’s work. When queried by Mr. Hony, Dr. Bing stated that he thought the chances were excellent, “about 8 to 2 odds,” that the research group could in fact develop the drug.

Dr. Bing further stated that he had found out that High Drugs, Hony’s only competitor for the type of product in question, had recently started developing essentially the same drug. He felt that working independently, there was a 7 out of 10 chance that High would succeed. Mr. Hony was concerned about the possibility that High would be able to develop the drug faster, thus obtaining an advantage in the market, but Dr. Bing assured him that Hony’s superior research capability made it certain that by starting development immediately, Hony would succeed in developing the drug before High. However, Dr. Bing pointed out that if Hony launched its development, succeeded, and marketed the drug, then High, by copying, would get its drug on the market at least as fast as if it had succeeded in an independent development.

Worried about the sales prospects of a drug so costly to develop, Mr. Hony talked to his marketing manager Mr. Margin, who said the market for the potential new drug depended on the acceptance of the drug by the medical profession and the share of the market Hony could capture. Mr. Hony asked Margin to make future market estimates for different situations, including estimates of future profits (assuming High entered the market shortly after Hony). Margin made the estimates shown below.

<table>
<thead>
<tr>
<th>Market Condition</th>
<th>Probability</th>
<th>Present Value of Profits ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Large Market Potential</td>
<td>.1</td>
<td>$500,000</td>
</tr>
<tr>
<td>Moderate Market Potential</td>
<td>.6</td>
<td>$250,000</td>
</tr>
<tr>
<td>Low Market Potential</td>
<td>3</td>
<td>$80,000</td>
</tr>
</tbody>
</table>

Mr. Hony was somewhat concerned about spending the $100,000 to develop the drug given such an uncertain market. He returned to Dr. Bing and asked if there was some way to develop the drug more cheaply or to postpone development until the market position was clearer. Dr. Bing said he would prefer his previous suggestion—an orderly research program costing $100,000—but that an alternative was indeed possible. The alternative plan called for a two-phase research program: an eight-month “low-level” phase costing $40,000, followed by a four-month “crash” phase costing $80,000. Dr. Bing did not think this program would change the chances of a successful product development. One advantage of this approach, Dr. Bing added, was that the company would know whether the drug could be developed successfully at the end of the eight-month period. The decision would then be made whether to undertake the crash program.

Mr. Hony further consulted Mr. Margin about the possibility that more market information would be available before making the decision to introduce the drug or to complete the crash development program if that strategy were
adopted. Mr. Margin stated that without a very expensive market research program, he would have no new information until well after the drug was introduced.

Mr. Hony inquired about the possibility of waiting until High’s drug was on the market and then developing a drug based on a chemical analysis of it. Dr. Bing said this was indeed possible and that such a drug could be developed for $50,000. However, Mr. Margin was dubious of the value of such an approach, noting that the first drugs out usually got the greater share of the market. He estimated the returns would only be about 50 percent of those given in the table, but that the cost of introducing the drug would be only $20,000; for the One-Phase or Two-Phase strategy, the cost of introducing the drug would be $50,000. Mr. Hony thought briefly about the possibility of going ahead with development after High had failed, but quickly realized that the chance of Hony’s success under such circumstances would be much too low to make the investment worthwhile. The chances of a successful development by High after Hony had failed in its development attempt were considered so remote (1 percent) that the “imitate-and-market” strategy was not considered once Hony had failed.

a. Draw the influence diagram for this case.

b. Draw the decision tree for this case.
This problem is also a little tricky to get into Supertree because of the variations in amount and timing of development costs and because of the variation in potential profits. The display below shows one way to get everything in properly via rewards, outcomes, and a simple endpoint model. There are two ways in which this representation differs from the tree shown above. First, the cost of 80 associated with Development Success in the Two Phase strategy has been included in the Profit node. Second, the costs associated with the Market node have been used as outcomes for that node.
Tree name: Hony Pharmaceuticals

<table>
<thead>
<tr>
<th>STRUCTURE</th>
<th>NAMES</th>
<th>OUTCOMES</th>
<th>PROBABILITIES</th>
<th>REWARDS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1D2 2 4 7</td>
<td>Strategy</td>
<td>One Phase Two Phase &amp; Wait Abandon</td>
<td></td>
<td>-100 -40 0 0</td>
</tr>
<tr>
<td>2C3 7</td>
<td>Development</td>
<td>Succeed Fail</td>
<td>.8 .2</td>
<td></td>
</tr>
<tr>
<td>3D5 7</td>
<td>Market</td>
<td>Depends on 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4C3 7</td>
<td>Competition</td>
<td>Succeed Fail</td>
<td>.7 .3</td>
<td>-50 0</td>
</tr>
<tr>
<td>5C6 6 6</td>
<td>Profit</td>
<td>Depends on 1</td>
<td>.1 .6 .3</td>
<td></td>
</tr>
<tr>
<td>6E</td>
<td>B$Market+Profit</td>
<td>Depends on 3 5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7E</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note that Supertree will still keep the numbers straight with the dependent outcomes, even though the abandon option does not lead to nodes 3 and 5.

c. Determine the best decision assuming risk indifference.

As illustrated in the partial tree display below, the One-Phase strategy is preferred with an expected value of $39,000.

<table>
<thead>
<tr>
<th>Strategy</th>
<th>OUTCOME, NODE 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>One Phase</td>
<td>-50 0</td>
</tr>
<tr>
<td>Two Phase</td>
<td>-50 0</td>
</tr>
<tr>
<td>Wait</td>
<td>-20 0</td>
</tr>
<tr>
<td>Abandon</td>
<td>0 0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Strategy</th>
<th>OUTCOME, NODE 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>One Phase</td>
<td>500 250 80</td>
</tr>
<tr>
<td>Two Phase</td>
<td>420 170 0</td>
</tr>
<tr>
<td>Wait</td>
<td>250 125 40</td>
</tr>
<tr>
<td>Abandon</td>
<td>0 0</td>
</tr>
</tbody>
</table>

Expected Value: 39

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Exp Val</th>
<th>Probs Development</th>
<th>Exp Val</th>
<th>Probs Competition</th>
<th>Exp Val</th>
</tr>
</thead>
<tbody>
<tr>
<td>One Phase</td>
<td>39</td>
<td>800 Succeed</td>
<td>74</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Two Phase</td>
<td>35</td>
<td>200 Fail</td>
<td>54</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wait</td>
<td>29</td>
<td>700 Succeed</td>
<td>42</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Abandon</td>
<td>0</td>
<td>300 Fail</td>
<td>0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

d. Draw the profit distribution for each alternative.
e. Determine the best decision assuming the payoff on each terminal node is certain and that Mr. Hony's risk tolerance is $100,000.

As illustrated below, a risk tolerance of $100,000 is enough to make the certain equivalents of the first two (more uncertain) strategies negative, making the Wait strategy preferred with a certain equivalent of $18,000.

Consider a clairvoyant so specialized that he could perfectly predict market outcomes, but nothing about development or competitive outcomes. Distinguish this case of partial perfect information (perfect information on only one of the uncertain variables) from the case of imperfect information.

f. Determine the value of market clairvoyance for a risk-indifferent (expected-value) decision-maker.

In the display below, the outcomes of profit are displayed as asterisks because the profits were input as dependent on strategy, and profit has now been moved in front of strategy. The proper way to interpret the outcomes
in this case, then, is as the large, moderate, and low market potentials listed in the problem description. Given large or moderate market potential, the preferred decision is the One-Phase strategy. Given low market potential, the preferred decision is to Abandon. The value of information on market potential is $62 - 39 = 23$, or $\$23,000$.

<table>
<thead>
<tr>
<th>Prob</th>
<th>Profit</th>
<th>Exp Val</th>
<th>Strategy</th>
<th>Exp Val</th>
</tr>
</thead>
<tbody>
<tr>
<td>.100</td>
<td>------</td>
<td>260</td>
<td>OnePhase</td>
<td>260</td>
</tr>
<tr>
<td>.600</td>
<td>------</td>
<td>60</td>
<td>TwoPhase</td>
<td>256</td>
</tr>
<tr>
<td>.300</td>
<td>------</td>
<td>0</td>
<td>Abandon</td>
<td>0</td>
</tr>
</tbody>
</table>

Expected Value: 62

Determining the value of market clairvoyance for a risk-averse decision-maker using Mr. Hony's utility. Assume that the cost of information is zero.

This tree display is the same as the one above, except for the use of a risk tolerance. Note that the risk aversion is sufficient to make Two-Phase the preferred decision in case of large market decision, while Abandon is still preferred in case of low market potential. The value of information $32 - 29 = 3$, or $\$3,000$, reflecting the expected value given up in the risk premium.

<table>
<thead>
<tr>
<th>Prob</th>
<th>Profit</th>
<th>Cert Eq</th>
<th>Strategy</th>
<th>Cert Eq</th>
</tr>
</thead>
<tbody>
<tr>
<td>.100</td>
<td>------</td>
<td>112</td>
<td>Wait</td>
<td>88</td>
</tr>
<tr>
<td>.600</td>
<td>------</td>
<td>42</td>
<td>TwoPhase</td>
<td>42</td>
</tr>
<tr>
<td>.300</td>
<td>------</td>
<td>0</td>
<td>Abandon</td>
<td>0</td>
</tr>
</tbody>
</table>

Certain Equivalent: 32

7.4 Mr. Able is the president of Blackgold, a petroleum distribution and marketing company that supplies refined products to a number of customers under long-term contracts at guaranteed prices. Recently, the price Blackgold must pay for petroleum has risen sharply. As a result, Blackgold is faced with a loss of $\$480,000$ this year because of its long-term contract with a particular customer.

Able has consulted his legal advisers to see if this supply contract might be relaxed in any way, and they have advised him that it contains a clause stating
that Blackgold may refuse to supply up to 10 percent of the promised amount because of circumstances beyond its control (a force majeure clause). Able’s marketing staff estimates that invoking the clause and selling the contested 10 percent at prevailing market prices would turn a loss of $480,000 into a net profit of $900,000.

However, the lawyers caution that the customer’s response to Blackgold’s invoking the clause is far from certain. The marketing staff claims there is a small chance that the customer will accept the invocation and agree to pay the higher price for the 10 percent. If it does not agree to pay the higher price, the lawyers feel it might sue for damages or it might simply decline to press the issue. In either case, Blackgold could then immediately sell the 10 percent on the open market at prevailing prices. A lawsuit would result in one of three possible outcomes: Blackgold loses and pays normal damages of $1,800,000; Blackgold loses and pays double damages of $3,600,000; and Blackgold wins. If it loses, it must also pay court costs of $100,000, but it need not deliver the oil.

a. Draw the influence diagram for Mr. Able’s problem.

The influence diagram for this problem is rather simple.

![Influence Diagram]

b. Structure the decision tree for Mr. Able’s problem.

The decision tree for Mr. Able’s problem is shown below.
This tree can easily be entered into Supertree. The endpoint values are entered using the Treevalue option for entering endpoint values directly.

This tree can easily be entered into Supertree. The endpoint values are entered using the Treevalue option for entering endpoint values directly.

The rewards and values shown above combine to give the correct endpoint values. See the tree display below for the final numbers (note that this tree is displayed with all rewards moved to the end of the tree.)

c. 

Assign dollar outcomes to all end points of the tree.

The rewards and values shown above combine to give the correct endpoint values. See the tree display below for the final numbers (note that this tree is displayed with all rewards moved to the end of the tree.)

Noting that invoking the clause could lead to a profitable outcome, Able has asked his staff to assess the likelihood of the various outcomes. After a great deal of discussion, they report that their best judgment indicates a one chance
in five the customer will agree to pay the market price for the contested 10 percent and, if it does not agree, a 50/50 chance it will sue Blackgold for damages. Based on past experience with cases of this type, the lawyers believe there is only a 10 percent chance of Blackgold winning the lawsuit, an 80 percent chance of losing normal damages, and a 10 percent chance of losing double damages.

d. Find the expected value of each of Blackgold’s alternatives.

<table>
<thead>
<tr>
<th>Clause</th>
<th>Exp Val</th>
<th>Probs Agree</th>
<th>Exp Val</th>
<th>Probs Sue</th>
<th>Exp Val</th>
<th>Probs Verdict</th>
<th>Exp Val</th>
</tr>
</thead>
<tbody>
<tr>
<td>Win</td>
<td>.100</td>
<td>900</td>
<td>.800</td>
<td>Lose</td>
<td>–1000</td>
<td>100 Win</td>
<td>900</td>
</tr>
<tr>
<td>Lose</td>
<td>.800</td>
<td>–1000</td>
<td>.500</td>
<td>Sue</td>
<td>–990</td>
<td>800 Lose</td>
<td>–1000</td>
</tr>
<tr>
<td>LoseDouble</td>
<td>.100</td>
<td>–2800</td>
<td>.500</td>
<td>NotSue</td>
<td>900</td>
<td>100 LoseDouble</td>
<td>–2800</td>
</tr>
<tr>
<td>NotAgree</td>
<td>.800</td>
<td>–45</td>
<td>.200</td>
<td>Agree</td>
<td>900</td>
<td>200 NotAgree</td>
<td>–45</td>
</tr>
<tr>
<td>Agree</td>
<td>.200</td>
<td>900</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

> Invoke: EV=144
> NotInvoke: EV=–480

Distressed by Mr. Able’s persistence in pursuing what they regard as an extremely risky course, the legal and marketing people propose two investigations with the hope of delaying any “reckless” actions. These include $2,000 for winning and dining the customer’s executives to sound out whether they might accept a revocation and $10,000 to have an objective outside survey team gather information on the possibilities of a lawsuit.
f. Determine the expected value of perfect information about whether Blackgold’s customers would agree to a price increase under the 10 percent clause and about whether they would sue if they did not agree to the increase. In each of these determinations, assume that only one of these two uncertainties can be resolved. How would you advise Able about the proposed studies?

As illustrated below, the preferred decision is to invoke the clause regardless of how the negotiations come out. Accordingly, the value of information is zero and wining and dining would be a waste of money (other than enjoying the meals).

As illustrated below, knowing whether or not the customer sues does change the preferred decision, and the value of information is $210 - 144 = 66$, or $66,000. However, whether the outside survey is worth the $10,000 depends on how accurately the survey can predict whether or not the customer sues (what can it give you for Nature’s Tree?)

One further investigation that could be conducted is a $15,000 study by an outside legal firm on the likelihood of the possible outcomes of the lawsuit.

g. Determine the expected value of perfect information about the outcome of a lawsuit (assuming this is the only uncertainty that can be resolved). How would you advise Able about the above study?

Perfect information on the verdict is worth $154 - 144 = 10$, or $10,000. Accordingly, the $15,000 study is not worth it.

h. Determine the expected value of perfect information (simultaneously) about all three uncertainties facing Blackgold.

The tree below illustrates the preferred decisions with perfect information on all three uncertainties. The value of simultaneous perfect information on all

---

**Expected Value: 144**

<table>
<thead>
<tr>
<th>Probs</th>
<th>Agree</th>
<th>Exp Val</th>
<th>Clause</th>
<th>Exp Val</th>
</tr>
</thead>
<tbody>
<tr>
<td>.800</td>
<td>NotAgree</td>
<td>–45</td>
<td>&gt;Invoke</td>
<td>–45</td>
</tr>
<tr>
<td>.200</td>
<td>Agree</td>
<td>900</td>
<td>&gt;Invoke</td>
<td>NotInvoke</td>
</tr>
</tbody>
</table>

**Expected Value: 210**

<table>
<thead>
<tr>
<th>Probs</th>
<th>Sue</th>
<th>Exp Val</th>
<th>Clause</th>
<th>Exp Val</th>
</tr>
</thead>
<tbody>
<tr>
<td>.500</td>
<td>Sue</td>
<td>–480</td>
<td>&gt;Invoke</td>
<td>–612</td>
</tr>
<tr>
<td>.500</td>
<td>NotSue</td>
<td>900</td>
<td>&gt;Invoke</td>
<td>NotInvoke</td>
</tr>
</tbody>
</table>

**Expected Value: 154**

<table>
<thead>
<tr>
<th>Probs</th>
<th>Verdict</th>
<th>Exp Val</th>
<th>Clause</th>
<th>Exp Val</th>
</tr>
</thead>
<tbody>
<tr>
<td>.100</td>
<td>Win</td>
<td>900</td>
<td>&gt;Invoke</td>
<td>–580</td>
</tr>
<tr>
<td>.800</td>
<td>Lose</td>
<td>140</td>
<td>&gt;Invoke</td>
<td>NotInvoke</td>
</tr>
<tr>
<td>.100</td>
<td>LoseDouble</td>
<td>–480</td>
<td>&gt;Invoke</td>
<td>NotInvoke</td>
</tr>
</tbody>
</table>
three uncertainties is $430 - 144 = 286$, or $\$286,000$! Note that the value of information is not additive—it can be either more or less than the sum of the individual values of information.

\[
\begin{array}{cccccc}
\text{Expected Value: 144} & & & & & \\
\text{Clause} & \text{Exp Val} & \text{Probs Agree} & \text{Exp Val} & \text{Probs Sue} & \text{Exp Val} & \text{Probs Verdict} & \text{Exp Val} \\
\text{Invoke} & 144 & .100 Win & 900 & .800 Lose & -1000 & .100 LoseDouble & -2800 \\
\text{Not Invoke} & -480 & .800 NotAgree & -45 & .500 Sue & -990 & .500 NotSue & 900 & .200 Agree & 900 \\
\end{array}
\]

7.5 The assumption behind the Black-Scholes formula is that stock prices perform "geometric Brownian motion." This implies that the probability distribution for the stock price $S$ at some future time, $t$, is lognormal:

\[
f(S) = \frac{1}{\sqrt{2\pi sS}} e^{-\frac{1}{2}\left(\frac{\ln(S) - m}{s}\right)^2}
\]

In this formula, $m$ and $s$ are not the mean and standard deviation of $S$, but rather of $\ln(S)$. For stock prices, $s$ is $s\sqrt{t}$, where $s$ is the stock’s volatility; in this problem, we reserve $s$ for the definitions below.

The mean, $m$, and variance, $v$ (square of standard deviation $s$) of $S$ are given by the formulas (see Problem 10–24)

\[
\begin{align*}
\mu &= e^{m + \frac{1}{2}s^2} \\
v &= e^{2m + 2s^2} - \mu^2
\end{align*}
\]

Show that the volatility, $s$, is approximately

\[
s = \frac{\sigma}{\mu} \left(1 - \frac{1}{4} \left(\frac{\sigma}{\mu}\right)^2 + \ldots \right)
\]

Hint: Use the approximations for $-1 < x < 1$:

\[
\ln(1 + x) = x - \frac{1}{2} x^2 + \ldots
\]

\[(1 \pm x)^n = 1 \pm nx + \ldots\]

From the equation for the mean, we can obtain

\[
\begin{align*}
\mu &= e^{m + \frac{1}{2}s^2} = e^m e^{s^2/2} \\
e^m &= \mu e^{-s^2/2}
\end{align*}
\]
From the equation for the variance, we obtain
\[ v = e^{2m+2s^2} - \mu^2 \]
\[ v = e^{2m} e^{2s^2} - \mu^2 \]
\[ v = (\mu e^{-s^2/2})^2 e^{2s^2} - \mu^2 \]
\[ v = \mu^2 e^{s^2} - \mu^2 \]
\[ v = \mu^2 (e^{s^2} - 1) \]

Because the standard deviation is the square of the variance, we can write
\[ \sigma^2 = \mu^2 (e^{s^2} - 1) \]
\[ (\sigma / \mu)^2 = e^{s^2} - 1 \]
\[ e^{s^2} = 1 + (\sigma / \mu)^2 \]
\[ s^2 = \ln(1 + (\sigma / \mu)^2) \]
\[ s = \sqrt{\ln(1 + (\sigma / \mu)^2)} \]

Using the approximation for the logarithm,
\[ s = \sqrt{(\sigma / \mu)^2 - (\sigma / \mu)^4 / 2 + \ldots} \]
\[ s = (\sigma / \mu) \sqrt{1 - (\sigma / \mu)^2 / 2 + \ldots} \]

Retain only the first two terms inside the square root, and use the expansion for the square root (n=1/2) to obtain
\[ s = (\sigma / \mu)(1 - (\sigma / \mu)^2 / 4 + \ldots) \]
Problems and Discussion Topics

8.1 Although the Dialog Decision Process has four distinct meetings, the first two meetings (Framing and Alternatives) are sometimes combined into one short meeting. This often occurs for the evaluation of Research and Development (R&D) projects. Why might this be true? What problems might arise in moving too quickly through these two meetings?

For R&D projects, the people involved are usually quite familiar with the context of the decision. The decision is usually a form of a Go/No Go decision or a Continue/Terminate decision. The policy decisions and the decision criterion should be well known. Therefore the Framing and Alternatives dialogs may not require two separate meetings.

However, the project leader should think carefully before shortening and combining these meetings. Sometimes R&D projects are done in relative isolation from the rest of the corporate environment; there may be connections between projects, initiatives, competitive pressures, strategic intent, and the like that need to be considered in framing the decision. In addition, taking the time to consider a rich set of alternatives (e.g., Continue, Increase Funding, Decrease Funding, Redirect Project,...) has been found to increase the value of R&D projects dramatically.

8.2 In high-level corporate strategy decisions, the Framing dialog can be the longest and most complex of the dialogs, and sometimes is broken into two meetings. Why might this be true? Why might the Alternatives dialog also be especially important in this situation?

High-level corporate strategy projects address a radical change in corporate direction. Mergers and acquisitions may be involved. Products and product lines may be promoted or discontinued. Production facilities may be constructed, remodeled, or closed. Business Units may be reorganized,
The number of possibilities are enormous, and reliable information hard to come by. A large business assessment effort may be needed before a shared understanding of the opportunities and challenges can be developed within the management team. The decision hierarchy may take a long time to develop, both in terms of policy (which in this case is close to the corporate vision statement) and the dimensions of the strategy table.

The Alternatives dialog can also be very important for high-level strategy decisions. The number of decision areas is large, the number of options in each decision area is large, and the number of possible combinations is truly overwhelming. Developing a small set of alternatives that are complete, coherent, and compelling is a task that requires time, effort, creativity, and ingenuity.

8.3 Consider some personal decision situation, either past or future, which involves several people. Examples of such decisions are a group choosing a restaurant for dinner, a couple deciding whether and when to get married, a class deciding on a class outing, and a family deciding on vacation plans.

a. Who should be on the project team?

b. Who should be on the decision board?

c. How should you involve the people who will be affected by the decision, but who are not on either team?

d. How do the answers to a, b, and c lead to decision quality? What potential decision quality problems could occur?

There are no set answers to this question. The purpose of the discussion topic is to think through the implications of the people side of the decision process. Issues of credibility, willingness to participate, time availability, differing interests and values, dominant and retiring personality types, and leadership should be part of the discussion.

8.4 For the following examples, estimate how much effort that should be devoted to the four parts of the decision process: framing, alternatives, analysis, decision. Express your answer in percentage of time/work in each part (four numbers adding to 1). Why might the proportion of effort vary?

a. Choice of a college or graduate school to attend.

b. Choice of a restaurant for a special (e.g., birthday or anniversary) dinner.

c. A life-changing decision situation such as marriage, choice of career, moving to a different area or country, etc.

d. Decision whether to buy a state lottery ticket when the prize has grown enormous because no one has won for several weeks.

e. Decision whether to attend a party on Monday night (e.g., Monday night football) or spend the evening studying for an exam Tuesday afternoon.
f. Choice between a fixed and variable-rate mortgage.
g. Choice of treatment in some life-threatening medical situation.

There are no set answers to this question, but there are some general expectations.

Framing will be relatively important if basic information is needed before you can even start thinking coherently about the decision; choice of a college might have this characteristic. Framing can also be important in situations where shared understanding and clear decision criteria are essential; life-changing decisions and medical decisions require special clarity in this area.

Creating and refining alternatives can be time consuming if there are multiple or complex possibilities. This can occur for any of the examples.

Analysis is often of less importance in personal decision-making than in business decision-making. However, choice of a mortgage deserves some analytical work, as well as spending more than a few dollars on lottery tickets.

The decision phase should not be slighted for important personal decisions. Time is required to come to grips with change and to accept its consequences.

8.5 In presenting a decision analysis, you often need to clearly and credibly present results to people who may not be familiar with or understand the methodology used to arrive at the results. In what other kinds of business situations is this also the case?

This situation is extremely common, occurring whenever the results from a technical section of an organization are presented to another section or to top management, when the analysis involves technical production problems and solutions, or when complicated finance is involved. Examples include presenting a new oil distribution strategy (with complicated pipeline and refinery balancing), presenting the results of an attempt to use recombinant DNA technology to produce a drug, presenting the results of developing a new computer chip, or even presenting new agricultural methods and hybrids to farmers. Perhaps a more important challenge for a decision analysis is that the results are often business recommendations made to people who are in other ways business experts—making it all the more crucial to establish credibility for a new decision analysis approach to their area of specialty.

8.6 List some of the considerations in deciding what level of detail to include in a decision analysis presentation.

Typical considerations are the audience (see problem 8.7), the purpose of the presentation, the stage of the analysis, how controversial the results are, the time allotted for the presentation, and the presentation medium. For instance, a presentation designed to introduce people to decision analysis and foster acceptance of it may contain more detail about methods than one to people who have already accepted it; the latter audience would probably want to focus more on conclusions and implications and less on technical subjects such as values of information and control. Further, an interim
presentation to a company project group will often include details of the model, assumptions, etc., because one of the purposes is to verify the decision basis with them and obtain their insight. Later and final presentations tend to focus on conclusions and on implementing the preferred alternative. Controversial results may also be better presented with more supporting detail, unless the audience is already familiar with the basis of the conclusions (such as when follow-up studies are presented). Finally, when time is shorter and projection media are used, less detail often helps to get the message across more effectively. To examine detailed results, the presenter might distribute printed copies of the materials being reviewed and request more time (two or three hours with breaks).

8.7 **How might you prepare a presentation differently if you were presenting to the operations research staff group as opposed to the vice president of marketing?**

Different audiences are used to seeing different levels of detail and require more or less of the “nuts and bolts” to establish credibility. An operations research staff group would probably welcome the details of model implementation and exponential utility functions (and, indeed, might be skeptical if not told the technical details), while a vice president of marketing would probably be bored and irritated at the “waste” of time—preferring to focus on the implied marketing strategy. Different information is relevant to different groups, and the wrong information can jeopardize acceptance and implementation.

8.8 **What kind of changes in procedures for making decisions might occur as a company adopts decision analysis? How would the number and function of people involved in decision-making change?**

In a typical company, decisions are made by the manager responsible for the particular area. The manager may or may not consult others (more consultation is required in matrix-management organizations) and may nor may not have to refer to a company forecast or spreadsheet model. However, the formal requirements rarely extend further than that.

Implementing decision analysis requires new and more formal procedures and more cooperation between different business areas. For instance, the board of directors would first of all have to agree on a risk tolerance and discount rate for use in all decisions. The company might even go so far as to designate specific experts in specific areas. (“Go talk to Irene Abernathy if you need a distribution on light pickup truck sales.”)

In addition, decision analysts are required at the company, whether in a central staff group or dispersed among different divisions. However, no matter what part of the organization the analysts work in, they require access to different divisions—purchasing, manufacturing, finance, marketing, and sales all must be considered before the net present value of a business opportunity can be calculated.

Perhaps the most sweeping changes in an organization are in the incentives required to make a decision analysis system work. To have the
incentive to make good decisions, employees must be assured that the quality of decisions—rather than outcomes—is evaluated. For managers used to being judged on the bottom line, this is a drastic change. However, given the built-in documentation of the decision basis provided by the analysis, this requirement is not as impossible as it might seem. Perhaps being freed of the vagaries of uncertainties outside their control would help managers sleep more easily.

8.9 Why might decision analysis have been adopted more rapidly in some industries than in others? Can all industries benefit from decision analysis?

Decision analysis has been applied most rapidly in those industries where the stakes, uncertainties, and complexities are high, and where a lot of money must be committed before any hints of success or failure are detected. For instance, in oil drilling or research and development, millions must be spent to drill a test well or fund a research project before the actual payoff (if any) begins. In contrast, the uncertainties and stakes are high in managing stock portfolios, but there are many (hourly) opportunities to buy in and sell out and much skepticism about predicting the future course of the market.

It is the authors’ belief that, given the flexibility of the level of analysis, almost all industries can obtain some benefit from decision analysis. Even an industry as faddish and unpredictable as the movie industry can use decision analysis to evaluate competing movie scripts (movie budget, potential box office receipts, etc.) or to decide when and how many test screenings to conduct before releasing a movie or even to evaluate different release and distribution strategies.

On the other hand, we do not mean to imply that decision analysis is applicable to all aspects of every business. For example, decision analysis would not be of much use for pro quarterbacks on the field—the situation simply changes too fast. However, it certainly could help with a bidding strategy for next year’s draft.

8.10 The Lone Star Drilling Company has several prospects in Oklahoma, Texas, and Louisiana. One of these prospects, in the state of Oklahoma, is called Moose Hill. A promising region for natural gas underlies the Moose Hill area at 20,000 feet. Gas discovered at this depth qualifies as “deep gas” and is allowed to sell at a free market price under current regulations. (This is a disguised version of an analysis performed in the late 1970s.) There is little chance of finding oil under Moose Hill.

For gas to be found, there must be a structural trap. Currently available seismic studies indicate 7 chances in 10 there will be a structural trap. Even with a trap, there is a good chance that the water saturation will be too high for a producing gas well. A producing well could yield between 2 and 25 MCF/day the first year (MCF = million cubic feet); yields over 30 MCF/day are unlikely. Over the 10-year life of the well, annual production is expected to decline by 20 to 25 percent per year.
Lone Star is currently drilling a well on the Moosejaw 1 section at Moose Hill. While that well is not expected to reach 20,000 feet for another year, it appears that the cost of drilling a 20,000-foot well will be $6 million to $10 million, plus about $2 million for completing the well if sufficient gas potential is found. Annual operating costs for similar wells run between $15,000 and $25,000.

Property in the area is divided into sections of one square mile. Operators with mineral leases within a given section usually pool together and drill one well per section. However, under Oklahoma’s forced-pooling statutes, any mineral leaseholder in a given section can decide to drill and invoke “forced pooling.” Holders of the remaining leases in the section must then either join in a drilling operation within 90 days, sharing proportionally in drilling costs and potential gas yield, or offer to sell their rights to the first leaseholder at a price set by the state. The purpose of the statute is to encourage drilling in Oklahoma.

Lone Star holds 99 percent of the mineral rights to Moosejaw 2, a section adjacent to Moosejaw 1. The holder of the other 1 percent of the mineral rights has invoked forced pooling. The state is in the process of setting a “fair” price.

If Lone Star decides to pool and drill on Moosejaw 2, it has the option of negotiating with another exploration company, Delta Resources, for a joint venture—proportional sharing of all future costs and revenues from the property. Delta Resources has expressed an interest in this joint venture opportunity.

Your group is to recommend the best courses of action for the Lone Star Drilling Company. As part of your presentation, include the following.

- What is the minimum amount of compensation Lone Star should accept to sell its current 99 percent share to the owner of the remaining 1 percent—assuming Lone Star must otherwise bear 99 percent of the costs of drilling (no joint venture with Delta Resources)?
- Assuming Lone Star decides to go ahead and drill, what joint venture share should it offer to Delta Resources?
- Assume the state sets $500,000 as the “fair” price for Lone Star’s interests in the lease. Calculate the expected value of perfect information on a few crucial uncertainties.

Make sure the presentation will be acceptable to, and understood by, the president of Lone Star, an old-time driller who never graduated from high school, but who has acquired considerable wealth, experience, and expertise over the years.

First of all, note that several data inputs that are not in the problem statement have been added to the spreadsheet below to allow a cash flow calculation: initial gas price and annual change in gas price, a royalty rate, and a tax rate. This information can be supplied when the problem is assigned. Another possibility is to let the student discover how to deal with missing information. The student could do some research and find the information (possibly in the
form of a range of values or of a probability distribution) or could make a reasonable estimate and emphasize that results are conditional on this estimate.

The following spreadsheet describes Moose Hill. All the input data is contained in the box at the top of the spreadsheet, and the entries in column C are the range names for the values in column B.

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
<th>J</th>
<th>K</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Moose Hill</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Description</td>
<td>Value Name</td>
<td></td>
<td>Units</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Initial Gas Flow</td>
<td>10 Flow_I</td>
<td>MCF/day</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Annual Change in Gas Flow</td>
<td>-23% Flow_gr</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Initial Gas Price</td>
<td>$4 Price_I</td>
<td>$/MCF</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Annual Change in Gas Price</td>
<td>6% Price_gr</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Annual Operating Cost</td>
<td>$15 Op_cost</td>
<td>$Kyr</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Drilling Cost</td>
<td>$8,000 Dr_cost</td>
<td>$K</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Cost to Complete Well</td>
<td>$2,000 Co_cost</td>
<td>$K</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>Royalties to Landowner</td>
<td>20% Royal</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>Lone Star Ownership</td>
<td>99% LS_own</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>Share to Delta Resources</td>
<td>0% Del_shr</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>Operating Days per Year</td>
<td>365 day_yr</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>Tax Rate</td>
<td>30% tax_rate</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>Operations</td>
<td>10% disc_rate</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>Calculations</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
| 17  | NPV | 19.2 ($ million, adjusted for shares)

The structure of the full decision tree is shown below.

**Tree name: Moose Hill**

<table>
<thead>
<tr>
<th>STRUCTURE</th>
<th>NAMES</th>
<th>OUTCOMES</th>
<th>PROBABILITIES</th>
</tr>
</thead>
<tbody>
<tr>
<td>1D2 8</td>
<td>Decision</td>
<td>Drill Sell</td>
<td></td>
</tr>
<tr>
<td>2D3 3 3 3</td>
<td>Del_Sh</td>
<td>0.25 .50 .75</td>
<td></td>
</tr>
<tr>
<td>3C4 9</td>
<td>Trap</td>
<td>Yes No</td>
<td>0.7 .3</td>
</tr>
<tr>
<td>4C5 9</td>
<td>Saturation</td>
<td>OK Bad</td>
<td>0.5</td>
</tr>
<tr>
<td>5C6 6 6</td>
<td>Flow_1</td>
<td>3 10 30</td>
<td>0.5 3 2</td>
</tr>
<tr>
<td>6C7 7 7</td>
<td>Price_gr</td>
<td>.03 .06 .11</td>
<td>.25 .50 .25</td>
</tr>
<tr>
<td>7E</td>
<td>Bc:\my documents\moose hill.xls$NPV$M</td>
<td>Depends on 2 5 6</td>
<td></td>
</tr>
<tr>
<td>8E</td>
<td>.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9E</td>
<td>B$-8*.99 *(1-Del_Sh)</td>
<td>Depends on 2</td>
<td></td>
</tr>
</tbody>
</table>
To obtain Lone Star’s minimum selling price, we can draw the first two nodes of the tree. The expected value of the zero share to Delta is Lone Star’s selling price: $2.22 million. The preferred share to Delta (Del_Shr) is visible at the same time: zero.

With the $500,000 price set by the state already in the tree, it is simple to calculate the values of information. The value of information on whether there is a structural trap is $4.75 - 2.22 = $2.53 million.

The value of information on whether the water saturation is too high (given that a trap has been found) is $6.43 - 2.22 = $4.21 million.
The value of information on the initial gas flow (given a structural trap and acceptable water saturation) is $2.57 million.

There is no value of information on the annual change in gas price. Note that the preferred decision is, in all cases, to drill.

8.11 Air Wars, Inc., a U.S. manufacturer of fighter planes, is aggressively marketing its popular Galaxy-MX and Scoop-UMI models to several emerging countries of the world. Sales discussions with two such countries, the Democratic Republic of Azultan (which has a reasonably stable government) and Byasfora’s new government (which is an uneasy coalition between the Leninist-Marxist wing and the rightist Christian Democrats) are in the final stages in early 1985. Both of these governments are also concurrently negotiating their air force armament needs with Le Mon Corporation, a European manufacturer. The discussions between the Le Mon Corporation and the governments of Azultan and Byasfora are of serious concern to the management of Air Wars, Inc.
Dr. Ian Winthrop, the CEO of Air Wars, Inc., has called an urgent meeting on the coming Saturday to assess Air Wars’ position and to develop a clear strategy to make these sales. Dr. Winthrop, in his memo to senior management, reaffirmed the urgency of the situation and called for their input during the weekend meeting. Dr. Winthrop stressed Air Wars’ commitment to growth during the coming years. He also brought senior management up to date on the key items in connection with the potential sale of the planes to Azultan and Byasfora.

- Air Wars’ Washington representative thought the U.S. government favorably regarded the plane sales to both the Azultan and Byasfora governments. However, the future stability of the new government in Byasfora was in question. A change in Byasfora’s government was likely to result in a much more extreme left-wing government supported by neo-communists, creating concern about a reversal of the U.S. government’s current support of the sale.

Probability of change of government in Byasfora by 1990: .45

The Washington representative also stressed the importance of the 1988 presidential election in relation to the sales to Byasfora. The most likely contender for president, if elected, is expected to consider the seriousness of the reported human rights violations in Byasfora and oppose the sale.

Probability of administration change in 1988: .80

Probability of opposition by new administration to Byasfora sale: .90

- The negotiations with the U.S. government on Air Wars’ cost structure for the sale of the Galaxy-MX and Scoop-UMi planes are nearing completion. Air Wars’ Finance Department projects the following prices in 1985 dollars (contingent upon three possible U.S. government positions on cost structure).

<table>
<thead>
<tr>
<th>Unit Price ($ million)</th>
<th>p=.30</th>
<th>p=.60</th>
<th>p=.10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Galaxy-MX</td>
<td>$3.9</td>
<td>$4.6</td>
<td>$5.2</td>
</tr>
<tr>
<td>Scoop-UMi</td>
<td>$2.65</td>
<td>$2.8</td>
<td>$3.15</td>
</tr>
</tbody>
</table>

- Last week, Dr. Winthrop met with the Secretary of State and the National Security Advisor. He was briefed on the current U.S. position with regard to the regional strategic balance of power in the Azultan and Byasfora area. As a result, Dr. Winthrop feels that the current administration is unlikely to approve the sales to
both the Azultan and Byasfora governments.

Probability of approving sales to both Azultan and Byasfora during 1987—1988: .01

- Recent discussions between Dr. Winthrop and both the Minister of Air Defense of Azultan and the General of Strategic Air Forces of Byasfora resulted in satisfactory agreement on the numbers of planes needed and a shipment schedule for each country:

<table>
<thead>
<tr>
<th>Shipment Schedule</th>
</tr>
</thead>
<tbody>
<tr>
<td>Galaxy Scoop Galaxy Scoop Galaxy Scoop Galaxy Scoop</td>
</tr>
<tr>
<td>Azultan 22   9  21   12  8   4  —   —</td>
</tr>
<tr>
<td>Byasfora —   —  18   13  22   8  11   8</td>
</tr>
</tbody>
</table>

In addition, the Azultan agreement calls for a five-year technology assistance contract at the rate of $185 million per year beginning in 1989; and the Byasfora agreement calls for a six-year $205 million per year technology assistance contract beginning in 1990. These technology assistance contracts would be terminated if the employees or assets of Air Wars were threatened by any future catastrophic sociopolitical change in these countries.

Probability of Catastrophic Sociopolitical Situation in Early 1990s
Azultan .10
Byasfora .45

- The Finance Department of Air Wars has completed reports on the creditworthiness of Azultan and Byasfora. The creditworthiness was found to be closely tied to the economic condition of these countries. These countries were also dependent on world economic conditions for a portion of their natural resource base revenues. Based on an analysis of world and domestic economic outlook, the probability of their being unable to finance the necessary portion of the sales amount and honoring the technical assistance contracts is as follows.

Probability of Defaulting on Payments After 1990
Azultan 30
Byasfora .25

The option of insuring the credit risk is being considered, and Air Wars is making confidential inquiries to determine the fees for such protection.

- Dr. Winthrop wants senior management to come up with a clear strategy for Air Wars and explain why it is the best strategy. In addition, Dr. Winthrop is also interested in trade-offs between
price and risks. In view of the Le Mon competition, Dr. Winthrop believes that price flexibility to achieve a competitive price is extremely important for closing the sale. Therefore, an analysis comparing Air Wars’ risks in relation to potential sales to Azultan and Byasfora is of significant value to Dr. Winthrop.

Your group is to prepare a presentation for Dr. Winthrop that addresses these concerns and clearly lays out the risks and possibilities inherent in this situation. The information given to you above may be redundant, incomplete, inconsistent, or unbelievable. Your group has to make the best of the situation and present a report. Your report should include:

a. A clear structuring of the uncertainties and their relation to one another and to Air Wars’ ultimate sales revenues

Below is one suggestion for putting the information into a spreadsheet. All the yeses and noes in the data input section are used in logical statements in the formulas in the “calculations” part of the spreadsheet to test whether revenues are lost in the current scenario. In the spreadsheet shown below, for example, plan A (selling to Azultan) was chosen, which zeroes out any Byasfora revenue. The plan was approved, or else all income would be zeroed out. Since there is turmoil in Azultan, the technical assistance revenue is lost after 1989. However, Azultan does not default, so the plane sales go through. If Azultan defaulted, those revenues would also be zeroed out after 1989. A similar set of tests go on for the Byasfora revenues.

The possible price levels have been put into an input table (the box labeled Price on the right), which uses a price key (Price_case) to set the sale prices to the low, medium, or high levels. This key is set by a single chance node in the tree. The numbers of planes and technical assistance revenues have also been put into the boxed time series in the middle of the spreadsheet. These time series are referred to directly by the logical tests in the calculations section. Finally, a 10 percent discount rate is used to determine the net present value.

Of course, constructing a similar spreadsheet may be a time-consuming task for students not familiar with these kinds of arithmetically simple but logically complicated calculations. Instructors may want to give hints and directions for approaching this problem and verify that students will have sufficient computer time to construct and debug their spreadsheets. See also the notes below regarding the tree for further information on potential difficulties.
Examples of the code used in the spreadsheet are, for cell C31, 
=INDEX($H$15:$J$15,Pr_case)*C20*(Plan="a")+(Plan="ab"){Approve="yes"},
and for cell D31, =INDEX($H$15:$J$15,Pr_case)*D20*(Plan="a")+(Plan="ab"){Approve="yes"}*(A_def="no"); the entry in D31 is copied across through J31. 

Cells D33:J33 and D37:J37 also contain the factor (A_turm="no").

The tree below presents one way of relating the uncertainties. It might be better to also include a “do nothing,” zero-value alternative at the decision node, but doing so makes the tree asymmetric—requiring three passes to the spreadsheet instead of one. While this is no real problem, three sets of file saves and recalls are required and the process is a little slower. However, a “do nothing” alternative is not realistic given the range of values associated with the other three alternatives—see the results presented below. Therefore, the “do nothing” alternative can be omitted without changing the results of the analysis.

The three sources of information on the chances of project approval have been incorporated in the dependent probabilities for node 4. Thus, approval
of plan A alone is a sure thing, chances of approval for B are reduced if a new administration gets into office, and the probability of change of government in Byasfora has been interpreted as the chances of approval should the administration not change. Accordingly, whether or not the administration changes is not an input to the spreadsheet after all, despite the place reserved for it in the data input section. Other interpretations are possible. The one disadvantage in structuring the information this way is that value of information on project approval cannot be calculated because the probabilities depend on the decision.

Finally, this form of the tree requires 288 spreadsheet calculations for the Evaluate step—somewhat more than the 250 endpoint evaluations allowed in the student version of Supertree. Instructors should therefore make sure that professional versions of Supertree are available or should restrict the tree size to one that requires fewer than 250 model calculations.

Tree name: Air Wars

<table>
<thead>
<tr>
<th>STRUCTURE</th>
<th>NAMES</th>
<th>OUTCOMES</th>
<th>PROBABILITIES</th>
</tr>
</thead>
<tbody>
<tr>
<td>1D 2 2</td>
<td>Plan</td>
<td>A B AB</td>
<td></td>
</tr>
<tr>
<td>2C 3 3</td>
<td>Price_case</td>
<td>1 2 3</td>
<td>.3 .6 .1</td>
</tr>
<tr>
<td>3C 4</td>
<td>Admin_chg</td>
<td>Yes No</td>
<td>.8 .2</td>
</tr>
<tr>
<td>4C 5</td>
<td>Approve</td>
<td>Yes No</td>
<td>Depends on 1 3</td>
</tr>
<tr>
<td>5C 6</td>
<td>A_turm</td>
<td>Yes No</td>
<td>.1 .9</td>
</tr>
<tr>
<td>6C 7</td>
<td>B_turm</td>
<td>Yes No</td>
<td>.45 .55</td>
</tr>
<tr>
<td>7C 8</td>
<td>A_def</td>
<td>Yes No</td>
<td>.3 .7</td>
</tr>
<tr>
<td>8C 9</td>
<td>B_def</td>
<td>Yes No</td>
<td>.25 .75</td>
</tr>
<tr>
<td>9E</td>
<td>Esc:\my documents\air wars.xls$NPV$M</td>
<td>Depends on 1 2 4 5 6 7 8</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Plan</th>
<th>Admin_chg</th>
<th>PROBABILITY, NODE 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Yes</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td>No</td>
<td>1.0</td>
</tr>
<tr>
<td>B</td>
<td>Yes</td>
<td>0.1 0.9</td>
</tr>
<tr>
<td></td>
<td>No</td>
<td>0.55 0.45</td>
</tr>
<tr>
<td>AB</td>
<td>Yes</td>
<td>0.01 0.99</td>
</tr>
<tr>
<td></td>
<td>No</td>
<td>0.01 0.99</td>
</tr>
</tbody>
</table>

b.  An analysis of the overall risk and the relative risk imposed by the individual uncertainties

Below are the probability distributions for the overall tree and for each uncertainty brought individually to the front of the tree.

As can be seen from the plots below, the plan to sell to Azultan alone overwhelms selling to Byasfora or selling to both, with an expected value of $714 million versus expected values of $97 million and $12 million, respectively. Note, however, that selling to Azultan alone has the potential of making up to only about $1,000 million, while selling to both could make up to $2,100 million (albeit with an extremely low probability).
In this tree the alternatives involve a set of uncertainties completely irrelevant to the preferred decision: the Byasfora uncertainties are irrelevant when you sell only to Azultan. Consequently, for instance, plotting the distributions for turmoil in Byasfora given that Azultan has been chosen would not reveal anything. To get around this problem, the distributions have been plotted with the various chance nodes brought to the very front of the tree, allowing the preferred decision to change if changes in the outcomes of the Byasfora uncertainties make it desirable.

In the plot with the sale price brought to the front of the tree (below) and plotted for each branch of Price_case, we see that changes in the allowed sale price vary the expected value by only $56 million and obviously do not change the preferred decision—the plot is virtually unchanged from the distribution seen for Plan A (above).
In the plot for change in administration (below), the expected value does
not change at all—showing that a possible change in the administration
neither affects the expected value of the preferred decision nor ever changes
the preferred decision.
Whether or not the chosen sale is approved (below) shifts the expected value from zero to $1,225 million. However, we must examine the reordered tree (which we will see later, in the value of information calculations) to see if the preferred decision changes.

As you might expect (given the $925 million technical assistance contract), turmoil in Azultan swings the expected value dramatically. Again,
we will need to examine the reordered tree to see whether the preferred decision changes.

Turmoil in Byasfora (below), of course does not change the expected value of selling to Azultan, nor does it change the preferred decision.
Azultan’s defaulting (below) has the second largest impact on the expected value. This uncertainty will be another one to check for changes in the preferred decision.

As we would expect, Byasfora’s default has no effect.
c. **Values of information for the most crucial uncertainties**

The first two nodes of the tree are shown below, illustrating the respective expected values and the effect on each of the price uncertainty.

<table>
<thead>
<tr>
<th>Prob</th>
<th>Price Case</th>
<th>Expected Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>.300</td>
<td>1</td>
<td>691.49</td>
</tr>
<tr>
<td>.600</td>
<td>2</td>
<td>719.52</td>
</tr>
<tr>
<td>.100</td>
<td>3</td>
<td>747.36</td>
</tr>
<tr>
<td>.300</td>
<td>1</td>
<td>93.63</td>
</tr>
<tr>
<td>.600</td>
<td>2</td>
<td>97.99</td>
</tr>
<tr>
<td>.100</td>
<td>3</td>
<td>102.43</td>
</tr>
<tr>
<td>.300</td>
<td>1</td>
<td>11.84</td>
</tr>
<tr>
<td>.600</td>
<td>2</td>
<td>12.35</td>
</tr>
</tbody>
</table>

As can be seen below, there is no value of information on the administration's price decision, although, as we saw earlier, price changes the expected value of the preferred alternative (and all the other alternatives). The value of control is \(747.36 - 713.89 = $33.47\) million.

<table>
<thead>
<tr>
<th>Prob</th>
<th>Price Case</th>
<th>Expected Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>.300</td>
<td>1</td>
<td>691.49</td>
</tr>
<tr>
<td>.600</td>
<td>2</td>
<td>719.52</td>
</tr>
<tr>
<td>.100</td>
<td>3</td>
<td>747.36</td>
</tr>
<tr>
<td>.300</td>
<td>1</td>
<td>93.63</td>
</tr>
<tr>
<td>.600</td>
<td>2</td>
<td>97.99</td>
</tr>
<tr>
<td>.100</td>
<td>3</td>
<td>102.43</td>
</tr>
<tr>
<td>.300</td>
<td>1</td>
<td>11.84</td>
</tr>
<tr>
<td>.600</td>
<td>2</td>
<td>12.35</td>
</tr>
</tbody>
</table>

There is no value of information on a change in administration, although such a change does affect the value of selling only to Byasfora by $230 million (by changing the probability of approval of a sale to Byasfora). There is no value of control.

<table>
<thead>
<tr>
<th>Prob</th>
<th>Admin Chg</th>
<th>Expected Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>.800</td>
<td>Yes</td>
<td>713.89</td>
</tr>
<tr>
<td>.200</td>
<td>No</td>
<td>713.89</td>
</tr>
</tbody>
</table>

There is no value of information on a change in administration, although such a change does affect the value of selling only to Byasfora by $230 million (by changing the probability of approval of a sale to Byasfora). There is no value of control.

<table>
<thead>
<tr>
<th>Prob</th>
<th>Admin Chg</th>
<th>Expected Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>.800</td>
<td>Yes</td>
<td>713.89</td>
</tr>
<tr>
<td>.200</td>
<td>No</td>
<td>713.89</td>
</tr>
</tbody>
</table>

Note that, with the current formulation of the tree, value of information on approval of the chosen sale cannot be calculated via the usual method because the probabilities depend on the alternative chosen. To obtain values of information by bringing the Approve node to the front of the tree, we would need to input a separate chance node for sale approval for each alternative.

However, by placing the approval node to immediately after the decision node, we can see that the value of an approved sale to Byasfora is still less than
the value of selling to Byasfora alone, and the value of selling to both (as you would expect) is the sum of the individual sales. Further, we can use this display to compute the values of information as if the Approve node were three separate nodes.

Bringing the node representing approval of sale to Azultan to the front of the tree would give us a choice among 713.89, 97.13, and 12.25, resulting in no change in the decision to sell to Azultan and no value of information. Bringing to the front of the tree the node for approval of sale to Byasfora only would give us a choice among 713.89, 511.19, and 12.25, resulting in no change in the decision to sell to Azultan alone and no value of information. Bringing the approval of sale to both node to the front of the tree would give a choice among 713.89, 97.13, and 1,225.90 (in case of approval), resulting in a decision to sell to both in that case. Since there is only a .01 chance of this occurring, the value with information is .99(713.89) + .01(1225.09) = 719.00. Accordingly, the value of information on approval of sale to both is 719 – 713.89 = $5.11 million—enough to do a little bit of Congressional polling.

Note also that, interestingly enough, only approval of sale to both has any value of control. The probability of 1.0 of selling to Azultan means that there is already effective control there; and being able to obtain approval of the sale to Byasfora does add enough value to overcome the value of the Azultan sale. For sale to both, however, the value of control is 1,225.09 – 713.89 = $511.2 million.

For turmoil in Azultan, there is no value of information. The value of control is 751.21 – 713.89 = $37.32 million.
For turmoil in Byasfora, there is no value of information and no value of control.

\[
\text{Expected Value: 713.89} \\
\text{Probs B\_tum Exp Val} \quad \text{Plan Exp Val}
\]
\[
\begin{array}{ccc}
\text{.450 Yes} & 713.89 & A \\
& & B \\
& & AB
\end{array}
\]
\[
\begin{array}{ccc}
\text{.550 No} & 713.89 & A \\
& & B \\
& & AB
\end{array}
\]

For the possibility that Azultan will default, there is no value of information. The value of control is $899.84 - 713.89 = $185.95 million.

\[
\text{Expected Value: 713.89} \\
\text{Probs A\_def Exp Val} \quad \text{Plan Exp Val}
\]
\[
\begin{array}{ccc}
\text{.300 Yes} & 280.01 & A \\
& & B \\
& & AB
\end{array}
\]
\[
\begin{array}{ccc}
\text{.700 No} & 899.84 & A \\
& & B \\
& & AB
\end{array}
\]

For the possibility that Byasfora will default, there is no value of information and no value of control.

\[
\text{Expected Value: 713.89} \\
\text{Probs B\_def Exp Val} \quad \text{Plan Exp Val}
\]
\[
\begin{array}{ccc}
\text{.250 Yes} & 713.89 & A \\
& & B \\
& & AB
\end{array}
\]
\[
\begin{array}{ccc}
\text{.750 No} & 713.89 & A \\
& & B \\
& & AB
\end{array}
\]

d. Recommendations for further study or for possible actions to manage the most serious risks.

As we have seen above, only the approval of sale to both Azultan and Byasfora has any value of information, although there are large values of control lurking about. However, gaining control over a government decision or the affairs of a foreign country is rather criminal.

Accordingly, the only recommended courses of action at this point would be to poll the administration about the possibilities of approval of a sale to both countries and perhaps to spend up to $185 million to guarantee that Azultan does not default on its payments (perhaps through some kind of financing subsidy). Other than these measures, there are no values of information to pursue and no measures of control that are legally obtainable.
Problems and Discussion Topics

9.1 Today’s newspaper probably carries several stories relating to decisions announced by a public figure or organization. Choose one of these decisions and describe the tasks that would need to have been done to make this a quality decision. Using your judgment as a member of the public, how would you rate the actual quality of this decision? Note that you do not have to agree with the decision to judge its quality.

None of the problems and discussion topics of this chapter have a fixed answer. Rather, they are intended to encourage thought about the quality of the decision-making process in several different arenas.

Public decision making poses a particular problem. Rarely is the public involved in the process, and so it is hard for individuals to judge the quality of the decision. However, it is our duty to come to some judgement on the quality of public decisions, if only at election time. Some relevant considerations might be:

Framing: Was the decision looked at in the appropriate local/state/national/global perspective? Was the input from all stakeholders systematically taken into account? Was sufficient time and attention given to the decision and the decision process? Is there some “hidden agenda?” Does the decision concern the true problem or is it a mask hiding some other problem?

Alternatives: Were several alternatives considered? Was a poor “straw man” alternative proposed so that it could be knocked down in favor of another alternative?

Information: Was there a fact-finding effort, or were arguments based on empty statements or beliefs? Were dependencies among the uncertainties addressed? Were intangibles (e.g., public image or prestige) overemphasized or underemphasized?
Values: How were long- and short-term results valued? Risk? The conflicting values of different sectors of the public? Difficult values such as the value of life?

Logic: Were things thought through to their conclusion? Were dynamic effects taken into account—social actions often have unintended results?

Commitment to Action: Was the decision taken seriously, with full intent for implementation, or was it like a campaign promise?

9.2 **R&D decisions are frequently framed as a basic Go/No Go or Continue/Stop decision for a specific project. What do you think is especially important to achieving quality in an R&D decision?**

Several aspects of R&D decisions that are different from other decisions:

Framing: Are all the stakeholders involved in the process? Research, marketing, finance, corporate strategy?

Alternatives: There are probably several very different alternatives besides “Continue.” These may arise from changes in the marketplace or from information gathered during the course of research.

Information: As a research project goes on, the information set should be periodically updated—market, technology, competition, probability of technical success.

Commitment to Action: Is there a strong linkage between those who will create the product/innovation and those who will produce it and those who will market it? Sometimes an R&D product continues long after the need for it has died.

9.3 **Corporate strategy decisions set the direction of the company for the next few years. Acquisitions, divestitures, and shutting down facilities may be part of the strategy. What do you think is especially important to achieving quality in a corporate strategy decision?**

Several aspects of corporate strategy decisions that are different from other decision:

Framing: Strategy decisions can be influenced by the “grass is greener on the other side of the hill” attitude. A prolonged period of business assessment should be an integral part of the framing. Has the perspective of all the stakeholders been represented in the process—employees, customers, vendors, stockholders, community?

Alternatives: There are normally an enormously large number of possible alternatives. These need to be reduced to a few before quantitative evaluation is possible. Time needs to be spent selecting alternatives that “fit” with the strengths and culture of the company, that are feasible, that will further the vision of the company.

Values: Are appropriate long- and short-term values in use? Are all the appropriate stakeholders’ values being taken into account?

Commitment to Action: Can the strategy be communicated to the organization? Will it be implemented successfully and enthusiastically?
9.4 Personal decisions such as choice of college, major, career, or marriage partner are decisions most of us face rarely, but which have great significance in our lives. Pick a personal decision that is (or will be) important to you. What do you think is especially important to achieving quality in this decision?

Personal decisions are really quite different from public or corporate decisions. Qualitative evaluation tends to be more important. Values are less clear. The following considerations may help:

Framing: Is time allotted for the decision to “sink in”? Have all the affected parties been included in the process? Does everyone agree that a decision needs to be made? Is the scope of the decision too wide or too narrow?

Alternatives: Has some creativity been used to come up with several attractive alternatives? Are all the alternatives doable?

Information: Is uncertainty being taken into account, even in a qualitative way? Are the interdependencies between factors taken into account?

Values: What is the decision criterion? This can be the most difficult question facing a decision-maker. In medical decision making, the patient may have to trade off pain, length of life, money, and consequences to the family and other loved ones. In life- or career-changing decisions, the decision maker realizes that, at least at a superficial level, the choice may affect his or her future value system.

Logic: Because personal decisions tend to be based on relatively qualitative reasoning, it is important to review the logic carefully, perhaps with someone not involved in the decision.

Commitment to Action: Many decisions go the way of New Year’s Day resolutions. Is there some mechanism to help make the choice really happen, whether it be an act of personal commitment or the choice of some external support?
10.1 State whether each of the following statements is or is not an event and if not, why not.

a. The temperature will be greater than 53°F in Bombay tomorrow.
   Yes. The intent of this question, of course, is for students to apply the clairvoyance test to determine if the clairvoyant can answer whether the proposed event occurred without having to ask for further information.

b. General Motors stock sold for more than $55 per share on the New York Stock Exchange.
   Yes, because this event occurred if General Motors stock has ever sold for more than $55 on the New York Exchange.

c. The weather is nice out today.
   No, because nice weather could mean vastly different things to different people.

d. The final version of the first printing of this book contains 1,034,246 characters.
   Yes.

e. Elm trees are taller than oak trees.
   No, because this event could refer to all the trees alive today or could refer to the average height of full-grown elm and oak trees, etc.

10.2 Draw the Venn diagram for the following events: 1,000 ≤ Units Sold < 5,000; Profit Margin per Unit < $2; Profit Margin per Unit ≥ $2. In which region(s) of
184 ANSWERS TO PROBLEMS AND DISCUSSION NOTES

the Venn diagram do the following events occur? (Profit = Units Sold · Profit Margin.)

a. Margin = $2 and Profit = $4,000

b. Margin = $1.50 and Profit = $2,000

c. Profit < $2,000 and Sales = 500 units

d. Profit = $15,000

10.3 The New England Patriots and the Cincinnati Bengals both have one game left in the season. They are not playing each other, and each game will go into overtime if necessary to produce a winner.

a. Draw the Venn diagram for this situation.

Note that this diagram is labeled differently from previous diagrams. This has been done for ease in identifying the relevant areas.
If the Patriots win and the Bengals lose, then the Patriots go to the playoffs. If the Patriots lose and the Bengals win, then the Bengals go to the playoffs. Otherwise, you are not sure who goes to the playoffs.

b. Show the region on the Venn diagram where you know the Patriots go to the play-offs for certain.

The Miami Dolphins also have one game left in the season and are playing neither the Patriots nor the Bengals. If Miami wins and both the Patriots and the Bengals lose, then the Patriots go to the playoffs. If Miami loses and both the Patriots and Bengals win, then the Patriots also go to the playoffs. Otherwise, the Miami game is not relevant.

c. Redraw the Venn diagram and show the regions where the Bengals go to the play-offs for sure.
10.4 For each of the following events or list of events, complete the list to make it mutually exclusive and collectively exhaustive.

a. The number of passenger automobiles assembled in the United States in 1985 was at least 20 million and less than 22 million.
   \[ \text{NPAAUS85} < 20 \text{ million} \]
   \[ \text{NPAAUS85} \geq 22 \text{ million} \]

b. The average length of all great white sharks reported caught off the coast of Australia is less than 15 feet.
   \[ \text{ALAGWSRCCA} < 15 \text{ feet} \]

b. The variable unit cost for producing the product is $1.75.
   \[ \text{VUCPP} < 1.75 \]
   \[ \text{VUCPP} > 1.75 \]

d. The market demand for adipic acid is at least 100 pounds per year and less than 150 million pounds per year. The market demand for adipic acid is greater than 400 million pounds per year.
   \[ \text{MDAA} < 100 \text{ lb/yr} \]
   \[ 150 \text{ MM lb/yr} \leq \text{MDAA} \leq 400 \text{ MM lb/yr} \]

e. A competitive product is introduced before our product is introduced.
   A competitive product is introduced at the same time our product is introduced.
   A competitive product is introduced after our product is introduced.
   A competitive product is introduced, but our product is not.
   A competitive product is not introduced.

f. Our market share is twice that of our nearest competitor.
   Our market share is less than twice that of our nearest competitor.
   Our market share is more than twice that of our nearest competitor.

10.5 A soldier is taking a test in which he is allowed three shots at a target (unmanned) airplane. The probability of his first shot hitting the plane is .4, that of his second shot is .5, and that of his third shot is .7. The probability of the plane’s crashing after one shot is .2; after two shots, the probability of crashing is .6; the plane will crash for sure if hit three times. The test is over when the soldier has fired all three shots or when the plane crashes.

a. Define a set of collectively exhaustive events.
   For this question, students can define a set of collectively exhaustive events on how many shots the soldier fires or on how many times the plane is hit.
The object is for the student to define a set that is collectively exhaustive but not mutually exclusive.

The plane is not hit.
The plane is hit at most once.
The plane is hit at most twice.
The plane is hit at most three times.

\[ b. \text{ Define a set of mutually exclusive events.} \]

The following set is mutually exclusive, but not collectively exhaustive.

The plane is hit once.
The plane is hit three times.

\[ c. \text{ Define a set of mutually exclusive and collectively exhaustive events.} \]

For this question, it is helpful for students to construct first the various sets of mutually exclusive and collectively exhaustive events and then to construct their combinations. These sets are:

The soldier fires one shot.
The soldier fires two shots.
The soldier fires three shots.

The plane is not hit.
The plane is hit on the first shot.
The plane is hit on the second shot.
The plane is hit on the first and second shot.
The plane is hit on the third shot.
The plane is hit on the first and third shots.
The plane is hit on the second and third shots.
The plane is hit on the first, second, and third shots.

The plane crashes.
The plane does not crash.

The joint events are as follows.

(one shot, first hit, crash)
(two shots, second hit, crash)
(two shots, first and second hit, crash)
(three shots, no hits, no crash)
(three shots, first hit, no crash)
(three shots, second hit, no crash)
(three shots, third hit, no crash)
(three shots, first and second hit, no crash)
(three shots, first and third hit, no crash)
(three shots, second and third hit, no crash)
(three shots, first and third hit, crash)
(three shots, second and third hit, crash)
(three shots; first, second, and third hit; no crash)
(three shots; first, second, and third hit; crash)
ANSWERS TO PROBLEMS AND DISCUSSION NOTES

d. What is the probability of the soldier shooting down the plane?

(One shot, first hit, crash) = .4 \times .2 = .080
(two shots, second hit, crash) = .6 \times .5 \times .2 = .060
(two shots, first and second hit, crash) = .4 \times .5 \times .6 = .120
(three shots, third hit, crash) = .6 \times .5 \times .7 \times .2 = .042
(three shots, first and third hit, crash) = .4 \times .5 \times .7 \times .6 = .084
(three shots, second and third hit, crash) = .6 \times .5 \times .7 \times .6 = .126
(three shots, three hits, crash) = .4 \times .5 \times .7 = .140

\[ \text{Total probability} = .080 + .060 + .120 + .042 + .084 + .126 + .140 = .652 \]

e. What is the mean number of shots required to shoot down the plane?

\[ \text{Mean shots} = (1 \times .08) + (2 \times (.06 + .12)) + (3 \times (.042 + .084 + .126 + .14)) = 1.62 \]

10.6 Define the joint events for the following two sets of events.

\[ m_1 = \text{Market Share < 5 percent} \]
\[ m_2 = 5 \text{ percent } \leq \text{ Market Share < 10 percent} \]
\[ m_3 = 10 \text{ percent } \leq \text{ Market Share} \]

\[ d_1 = \text{Development Cost } \leq \$2 \text{ million} \]
\[ d_2 = \$2 \text{ million } < \text{ Development Cost } \leq \$5 \text{ million} \]
\[ d_3 = \$5 \text{ million } < \text{ Development Cost} \]

The joint events are constructed by taking all combinations of the two sets of events.

\[ (m_1, d_1) \]
\[ (m_1, d_2) \]
\[ (m_1, d_3) \]
\[ (m_2, d_1) \]
\[ (m_2, d_2) \]
\[ (m_2, d_3) \]
\[ (m_3, d_1) \]
\[ (m_3, d_2) \]
\[ (m_3, d_3) \]

10.7 For the data in the preceding problem, assume that the events have been approximated by discrete values as follows.

\[ m_1 = \text{Market Share = 3 percent} \]
\[ m_2 = \text{Market Share = 7 percent} \]
\[ m_3 = \text{Market Share = 13 percent} \]

\[ d_1 = \text{Development Cost = } \$1.5 \text{ million} \]
\[ d_2 = \text{Development Cost = } \$3 \text{ million} \]
\[ d_3 = \text{Development Cost = } \$7 \text{ million} \]

a. Define the joint set of events.
The joint set of events is constructed by taking all combinations of the events.

\[(m_1, d_1)\]
\[(m_1, d_2)\]
\[(m_1, d_3)\]
\[(m_2, d_1)\]
\[(m_2, d_2)\]
\[(m_2, d_3)\]
\[(m_3, d_1)\]
\[(m_3, d_2)\]
\[(m_3, d_3)\]

b. If Market Size \(= \$100\) million and Revenue \(= (\text{Market Size} \cdot \text{Market Share}) - \text{Development Cost}\), calculate the Revenue for each joint event.

For each of the joint events, Revenue is easily calculated.

\[(m_1, d_1) = (100 \cdot 0.03) - 1.5 = 1.5\]
\[(m_1, d_2) = (100 \cdot 0.03) - 3 = 0\]
\[(m_1, d_3) = (100 \cdot 0.03) - 7 = -4\]
\[(m_2, d_1) = (100 \cdot 0.07) - 1.5 = 5.5\]
\[(m_2, d_2) = (100 \cdot 0.07) - 3 = 4\]
\[(m_2, d_3) = (100 \cdot 0.07) - 7 = 0\]
\[(m_3, d_1) = (100 \cdot 0.13) - 1.5 = 11.5\]
\[(m_3, d_2) = (100 \cdot 0.13) - 3 = 10\]
\[(m_3, d_3) = (100 \cdot 0.13) - 7 = 6\]

For each of the joint events, Revenue is easily calculated.

\[(m_1, d_1) = (60 \cdot 0.03) - 1.5 = 0.3\]
\[(m_1, d_2) = (60 \cdot 0.03) - 3 = -1.2\]
\[(m_1, d_3) = (60 \cdot 0.03) - 7 = -5.2\]
\[(m_2, d_1) = (60 \cdot 0.07) - 1.5 = 2.7\]
\[(m_2, d_2) = (60 \cdot 0.07) - 3 = 1.2\]
\[(m_2, d_3) = (60 \cdot 0.07) - 7 = -2.8\]
\[(m_3, d_1) = (60 \cdot 0.13) - 1.5 = 6.3\]
\[(m_3, d_2) = (60 \cdot 0.13) - 3 = 4.8\]
\[(m_3, d_3) = (60 \cdot 0.13) - 7 = 0.8\]

10.8 On the air route between Chicago and Los Angeles, there is either a head wind or tail wind. Depending on which way the wind is blowing and how fast, flights from Chicago to Los Angeles may be either early, on time, or late. We define the following events.

\(w_1\) — Head Wind
\(w_2\) — Tail Wind
The joint probabilities are as follows:

\[ w_1 \text{ and } a_1 = 0.06 \]
\[ w_1 \text{ and } a_2 = 0.12 \]
\[ w_1 \text{ and } a_3 = 0.22 \]
\[ w_2 \text{ and } a_1 = 0.39 \]
\[ w_2 \text{ and } a_2 = 0.18 \]
\[ w_2 \text{ and } a_3 = 0.03 \]

a. What is the marginal probability of a head wind?

The marginal probability is the sum of the probabilities for the joint events that contain the event in question.

\[ 0.06 + 0.12 + 0.22 = 0.40 \]

b. What are the conditional probabilities for arriving early, on time, or late given a tail wind?

The probabilities for each of the events is calculated by dividing the appropriate joint probability by the appropriate marginal probability.

\[ p(a_1 \mid w_2, S) = 0.39 / 0.60 = 0.65 \]
\[ p(a_2 \mid w_2, S) = 0.18 / 0.60 = 0.30 \]
\[ p(a_3 \mid w_2, S) = 0.03 / 0.60 = 0.05 \]

c. Given you arrive on time, what is the probability you had a tail wind? If you arrive early? If you arrive late?

The procedure for this question is identical to the procedure used in 10.8b.

\[ p(w_2 \mid a_2, S) = 0.18 / 0.30 = 0.60 \]
\[ p(w_2 \mid a_1, S) = 0.39 / 0.45 = 0.87 \]
\[ p(w_2 \mid a_3, S) = 0.03 / 0.25 = 0.12 \]

10.9 The Surprise Dog man at Fenway Park sells all his hotdogs for the same price, but he does not tell you in advance what you are getting. You could receive a regular dog or foot long dog, either of which could be a cheese or chili dog. We define the following events.

\[ l_1 = \text{You get a foot-long dog} \]
\[ l_2 = \text{You get a regular dog} \]
\[ c_1 = \text{You get a cheese dog} \]
\[ c_2 = \text{You get a chili dog} \]

The marginal probability of getting a foot long dog is 0.25. The probability of getting a foot long chili dog is 0.225, and the probability of getting a regular cheese dog is 0.45.
a. What is the marginal probability of getting a cheese dog?
The following numbers can be obtained from completing the tree.
\[ p(c_1 \mid S) = .45 + (.1 \times .25) = .475 \]
b. What is the probability of getting a regular chili dog? The following can also be read off the tree.
\[ p(l_2, c_1 \mid S) = .30 \]

10.10 A weather forecaster said that San Francisco and Los Angeles have probabilities of .7 and .4, respectively, of having rain during Christmas day. She also said that the probability of their both having rain is .28.

a. Find the probability of rain in San Francisco on Christmas day given there is rain in Los Angeles on Christmas day.
Completing the tree for this problem (you will need to do part of one tree first, then add the other marginal probability to the flipped tree to complete it) shows that the probabilities of rain are independent. Accordingly, the probability of rain in San Francisco is the same regardless of whether there is rain in Los Angeles: .7.

b. Find the probability of rain in Los Angeles on Christmas day given rain in San Francisco on Christmas day.
Still .4.

c. Find the probability of rain in San Francisco or Los Angeles (or both) on Christmas day.
Only the no rain in either city branch is omitted, so .28 + .42 + .12 = .82.

10.11 Your resident expert on Soviet deployments, Katyusha Caddell, has just given you his opinion on recent Soviet missile developments. The Soviets are building silos which may be of type 1 or type 2 (it is too early to tell), and Katyusha is unsure about which of two possible missile types the Soviets will be deploying in them. He describes the following events.

\[ s_1 — Si\text{lo of type 1 built} \]
\[ s_2 — Si\text{lo of type 2 built} \]
\[ m_1 — Type 1 missile deployed \]
\[ m_2 — Type 2 missile deployed \]

Katyusha puts the probability of the silos being type 2 at .6 and figures that type 2 silos mean a .7 probability of type 1 missiles, while type 1 silos mean a .8 probability of type 2 missiles. He further puts the marginal probability of type 2 missile deployment at .6. Do the marginal probabilities agree?
No. Using all the information except the marginal probability of type 2 missiles allows completion of the tree with silos coming first. Reversing that
tree produces marginal probabilities of .5, .5, not the .4, .6 indicated by Katyusha.

10.12 You and a friend are pondering buying tortilla chips and salsa at a baseball game. Your friend tells you he has made a systematic study of the different varieties of salsa and says the possible types are salsa tomatillo, salsa fresca, and traditional salsa. (The workers at the snack bar do not know what kind it is.) Furthermore, the salsa could be hot (spicy) or not hot. Your friend makes the following predictions.

The chance of hot salsa tomatillo is .08.
The chance of hot salsa fresca is .15.
The chance of not hot traditional salsa is .18.
The chance of getting salsa fresca is .3 and of getting traditional salsa is .6.

a. What is the probability of getting not hot salsa tomatillo?
The probability is .02, as shown in the tree below.

<table>
<thead>
<tr>
<th>Probs Salsa</th>
<th>Probs Spiciness</th>
<th>Joint Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>.100 Tomatillo</td>
<td>.800 Hot</td>
<td>.08</td>
</tr>
<tr>
<td>.300 Fresca</td>
<td>.500 Hot</td>
<td>.15</td>
</tr>
<tr>
<td>.600 Traditional</td>
<td>.700 Hot</td>
<td>.42</td>
</tr>
</tbody>
</table>

b. What is the conditional probability that the salsa is hot, given that it is salsa tomatillo?
See the above tree. The probability is .80.

c. What is the marginal probability that the salsa is hot?
As shown in the tree below, the probability is .65.

<table>
<thead>
<tr>
<th>Probs Spiciness</th>
<th>Probs Salsa</th>
<th>Joint Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>.650 Hot</td>
<td>.123 Tomatillo</td>
<td>.08</td>
</tr>
<tr>
<td>.350 NotHot</td>
<td>.429 Fresca</td>
<td>.15</td>
</tr>
<tr>
<td>.514 Traditional</td>
<td>.057 Tomatillo</td>
<td>.02</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Probs Salsa</th>
<th>Probs Spiciness</th>
<th>Joint Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>.650 Hot</td>
<td>.123 Tomatillo</td>
<td>.08</td>
</tr>
<tr>
<td>.350 NotHot</td>
<td>.429 Fresca</td>
<td>.15</td>
</tr>
<tr>
<td>.514 Traditional</td>
<td>.057 Tomatillo</td>
<td>.02</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Probs Spiciness</th>
<th>Probs Salsa</th>
<th>Joint Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>.650 Hot</td>
<td>.123 Tomatillo</td>
<td>.08</td>
</tr>
<tr>
<td>.350 NotHot</td>
<td>.429 Fresca</td>
<td>.15</td>
</tr>
<tr>
<td>.514 Traditional</td>
<td>.057 Tomatillo</td>
<td>.02</td>
</tr>
</tbody>
</table>

10.13 Frequently, people use tests to infer knowledge about something. A current (controversial) example is the use of a blood test to see if a person has the AIDS virus or not. The test results reflect current knowledge of the virus' characteristics, and test accuracy may be a matter of concern. How should the information
represented by the blood test result be used to update knowledge of the test subject’s condition? Bayes’ Rule gives the answer to this question.

Suppose a number of people have taken an XYZ virus test with the following results. (The numbers are purely illustrative and are not intended to reflect current understanding of the AIDS blood test.)

<table>
<thead>
<tr>
<th>Subject’s Condition</th>
<th>Test Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>XYZ Virus</td>
<td>Positive</td>
</tr>
<tr>
<td></td>
<td>.99</td>
</tr>
<tr>
<td></td>
<td>Negative</td>
</tr>
<tr>
<td></td>
<td>.01</td>
</tr>
<tr>
<td>No XYZ Virus</td>
<td>Positive</td>
</tr>
<tr>
<td></td>
<td>.95</td>
</tr>
<tr>
<td></td>
<td>Negative</td>
</tr>
<tr>
<td></td>
<td>.10</td>
</tr>
</tbody>
</table>

If a person has taken this test and the result turns out to be positive, what is the probability that he or she does not have the XYZ virus?

(Testing for the AIDS virus also involves serious issues of rights to privacy and due process. This problem addresses only the information gained by using a test where the outcome of the test is not a perfect indicator of the underlying condition.)

The probabilities can be entered into Supertree in the order they are given. Because only the probabilities are of interest, the endpoint has been entered as zero. When the tree is displayed in reverse order, we find the following set of probabilities.

<table>
<thead>
<tr>
<th>Probs Test Result</th>
<th>Exp Val</th>
<th>Probs Condition</th>
<th>Exp Val</th>
</tr>
</thead>
<tbody>
<tr>
<td>.343 XYZ</td>
<td>0</td>
<td>.657 No XYZ</td>
<td>0</td>
</tr>
<tr>
<td>.145 Positive</td>
<td>0</td>
<td>.001 XYZ</td>
<td>0</td>
</tr>
<tr>
<td>.855 Negative</td>
<td>0</td>
<td>.999 No XYZ</td>
<td>0</td>
</tr>
</tbody>
</table>

Amazingly enough, even a test that looks as good on this one on a historical analysis only gives a 34 percent chance of having the virus with a positive test result. However, note the nearly definitive negative result—perhaps a reason for following up a positive test result with a different test.

10.14 Your professor tells you that only 50 percent of the students in her class will do the homework and pass the class; 25 percent will not do the homework and will still pass the class; 8.3 percent will do the homework and study too much
ANSWERS TO PROBLEMS AND DISCUSSION NOTES

(missing sleep) and still pass. The professor figures 30 percent will not do the homework, 60 percent will do the homework, and 10 percent will work too much.

According to the professor, are doing the homework and passing the class probabilistically dependent or independent?

If you start to complete the tree, you will find that \(0.8333 = 0.5/0.6 = 0.25/0.30 = 0.083/0.1\), which means that the conditional probability of passing is the same regardless of how much homework the student does, and that passing and doing the homework are probabilistically independent.

10.15 You suspect that your corns hurt when your mother is about to call you. However, you think that the chance of getting a call from your mother and your corns not hurting is about 0.5. Your corns hurt about 10 percent of the time.

What is the marginal probability of your mother calling if her calling and your corns hurting are probabilistically independent?

The probability is simply calculated as

\[ (1-0.10) \times 0.5 = 0.556 \]

10.16 Use the information from problem 10.7 to perform the following calculations.

a. Formulate the joint events and calculate the probabilities and revenues for them. Assume probabilities of 0.25, 0.50, and 0.25 for \(m_1\), \(m_2\), and \(m_3\) and for \(d_1\), \(d_2\), and \(d_3\), respectively.

Using Supertree, we find the following:

<table>
<thead>
<tr>
<th>Probs</th>
<th>Share</th>
<th>Exp Val</th>
<th>Probs</th>
<th>Cost</th>
<th>Exp Val</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>0.03</td>
<td>-0.63</td>
<td>0.50</td>
<td>3</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.25</td>
<td>7</td>
<td>-4.00</td>
</tr>
<tr>
<td>0.50</td>
<td>0.07</td>
<td>3.38</td>
<td>0.50</td>
<td>3</td>
<td>4.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.25</td>
<td>7</td>
<td>0.00</td>
</tr>
<tr>
<td>0.25</td>
<td>0.13</td>
<td>9.38</td>
<td>0.50</td>
<td>3</td>
<td>10.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.25</td>
<td>7</td>
<td>6.00</td>
</tr>
</tbody>
</table>

b. Plot the cumulative probability distribution for revenue.
c. Plot the histogram for revenue. (Choose the bin size to give a good representation of the data.)
d. Calculate the mean, variance, and standard deviation of the distribution.
Mean: 3.875; variance: 16.92; standard deviation: 4.11.

10.17 The annual revenues from a new gasoline additive depend on annual U.S. gas consumption and on the average price of gasoline over the next year. It is estimated that 1 bottle of additive will be sold for every 1,000 gallons of gasoline consumed. The price will be set at twice the price for a gallon of gas. Discretized estimates of U.S. gas consumption next year put a .3 chance on consumption being 1 billion gallons, a .6 chance on consumption being 1.5 billion gallons, and a .1 chance on consumption being 2 billion gallons. Similarly, average gas prices have a .25 chance of being $0.50, a .5 chance of being $1.00, and a .25 chance of being $1.25.

a. Formulate a probability tree for revenue.

<table>
<thead>
<tr>
<th>Tree name: Gas Additive</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>STRUCTURE NAMES</strong></td>
</tr>
<tr>
<td>1C2 2 2 Usage</td>
</tr>
<tr>
<td>2C3 3 3 Price</td>
</tr>
<tr>
<td>3E B$2^&quot;Price*Usage&quot;</td>
</tr>
</tbody>
</table>

Note that since usage is in billions of gallons and consumption of the additive will be one per thousand gallons, this tree will produce results in millions of dollars.
b. Calculate the probabilities and revenues for the joint events.

The revenues and probabilities are shown in the tree below. The column labeled Probabilities has been added to the display.

```
<table>
<thead>
<tr>
<th>Probs Usage</th>
<th>Exp Val</th>
<th>Probs Price</th>
<th>Exp Val</th>
<th>Probabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>.250</td>
<td>.5</td>
<td>1.00</td>
<td>.075</td>
<td></td>
</tr>
<tr>
<td>.500</td>
<td>1</td>
<td>2.00</td>
<td>.150</td>
<td></td>
</tr>
<tr>
<td>.250</td>
<td>1.25</td>
<td>2.50</td>
<td>.075</td>
<td></td>
</tr>
<tr>
<td>.300</td>
<td>1</td>
<td>1.88</td>
<td>.150</td>
<td></td>
</tr>
<tr>
<td>.250</td>
<td>.5</td>
<td>1.50</td>
<td>.075</td>
<td></td>
</tr>
<tr>
<td>.500</td>
<td>1</td>
<td>3.00</td>
<td>.300</td>
<td></td>
</tr>
<tr>
<td>.250</td>
<td>1.25</td>
<td>3.75</td>
<td>.150</td>
<td></td>
</tr>
<tr>
<td>.600</td>
<td>1.5</td>
<td>2.81</td>
<td>.025</td>
<td></td>
</tr>
<tr>
<td>.500</td>
<td>1</td>
<td>4.00</td>
<td>.050</td>
<td></td>
</tr>
<tr>
<td>.250</td>
<td>1.25</td>
<td>5.00</td>
<td>.025</td>
<td></td>
</tr>
</tbody>
</table>
```

Expected Value: 2.63

```
<table>
<thead>
<tr>
<th>Revenue</th>
<th>Cumulative Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>1.00</td>
<td>0.10</td>
</tr>
<tr>
<td>2.00</td>
<td>0.20</td>
</tr>
<tr>
<td>3.00</td>
<td>0.30</td>
</tr>
<tr>
<td>4.00</td>
<td>0.40</td>
</tr>
<tr>
<td>5.00</td>
<td>0.50</td>
</tr>
<tr>
<td>6.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>
```

c. Plot the cumulative probability distribution for revenue.

d. Plot the histogram for revenue.
Note that the value in the range 0 to 1 is caused by a value of 1 in the tree—a value is in the range if it is greater than the lower bound of the range and less than or equal to the upper bound of the range.

e. **Calculate the mean, variance, and standard deviation of the distribution.**
Mean: 2.65; variance: 0.925; standard deviation: 0.9618.

10.18 You are offered an opportunity to engage in a series of three coin flips (.5 probability of winning or losing). For the first flip, you would bet $1 and either double your money or lose it. For the second flip (if you had won the first flip), you would reinvest your $2 and either double your money or lose it; if you had lost the first flip, you would bet another $1 and double or lose it. The process is repeated for the third coin flip, with having either the money you won on the second flip or a new $1 investment if you lost the second flip.

a. **Draw the probability tree for the three coin flips.**
b. Calculate your winnings or losses for each joint event and the associated probabilities.
Each joint event has a probability of .125.

c. Plot the cumulative probability distribution for your proceeds from the flips (wins or losses).

d. Plot the histogram for your proceeds from the flips.
e. Calculate the mean, variance, and standard deviation for your proceeds from the flips.
Mean: 0; variance: 9; standard deviation: 3.

10.19 Explain graphically why the following relationships are true for the events I and Ø:

\[ I \text{ and } Ø = Ø \]
\[ I \text{ or } Ø = I \]

Use these relationships and the probability axioms to prove the following probability:

\[ p(Ø | S) = 0 \]

Here is a plot of I on the same diagram as Ø. (I and Ø) is overlap between the two. There is not any overlap, which is the same as Ø.
Similarly, \( I \) or \( \emptyset \) is the area occupied by \( I \) or \( \emptyset \). Since \( \emptyset \) occupies no area and \( I \) is all of itself, this is the same thing as \( I \).

\[
I \text{ and } \emptyset = \emptyset 
\]
from above

\[
p(I \text{ or } \emptyset | S) = p(I|S) + p(\emptyset | S) 
\]
from equation 10-12

\[
I \text{ or } \emptyset = 1 
\]
from above

\[
p(I | S) = p(I | S) + p(\emptyset | S) 
\]
substituting in

\[
p(I | S) = 1 
\]
from equation 10-11

\[
1 = 1 + p(\emptyset | S) 
\]
substituting in

\[
0 = p(\emptyset | S) 
\]
Q.E.D.

10.20 Let \( A \) and \( B \) be any two events and let \( A' = \text{not } A \). Explain graphically why the following relations are true:

\[
A \text{ or } B = A \text{ or } (A' \text{ and } B) 
\]

\[
B = (A \text{ and } B) \text{ or } (A' \text{ and } B) 
\]

\[
A \text{ and } (A' \text{ and } B) = \emptyset 
\]

Use these relationships, the results of problem 10.19, and the probability axioms to prove the following relationship among probabilities:

\[
p(A \text{ or } B | S) = p(A | S) + p(B | S) - p(A \text{ and } B | S) 
\]

The events are shown in the diagram below.
202  ANSWERS TO PROBLEMS AND DISCUSSION NOTES

\[ A' \]

\[ A \]

\[ B \]

\( (A' \text{ and } B) \) is the lighter part of the oval on the right. \((A \text{ or } (A' \text{ and } B))\) is the area of \(A\) plus the area of \((A' \text{ and } B)\). The result is the area of \(A\) plus the non-overlapping area of \(B\), with is the same as \((A \text{ or } B)\) by equation 10-16.

\( (A \text{ and } B) \) is the darker overlap of the two ovals. \((A' \text{ and } B)\) is the lighter part of the oval on the right. \((A \text{ and } B) \text{ or } (A' \text{ and } B)\) is the sum of these two areas, which is the same as \(B\).

\( A \) is the black and darker (overlap) part of the oval on the right. \((A' \text{ and } B)\) is the lighter part of the oval on the right. There is no overlap between these areas, so \((A \text{ and } (A' \text{ and } B))\) is \(\emptyset\).

\[
\text{if } (A \text{ and } B) = \emptyset \text{ then } p(A \text{ and } B \mid S) = 0 \quad \text{from problem 10-19}
\]

\[
p(A \text{ or } B \mid S) = p(A \mid S) + p(B \mid S) \quad \text{from problem 10-12}
\]

10.21  Let \(A\) and \(B\) be any two events and let \(B' = \text{not } B\). Explain graphically why the following relationships are true:

\[
A = (A \text{ and } B) \text{ or } (A \text{ and } B')
\]

\[
\emptyset = (A \text{ and } B) \text{ and } (A \text{ and } B')
\]

Use these relationships and the probability axioms to prove the following simple application of the Expansion Theorem:

\[
p(A \mid S) = p(A,B \mid S) + p(A,B' \mid S)
\]

These events are shown in the diagram below.

\[ A' \]

\[ A \]

\[ B \]

\( (A \text{ and } B) \) is the darker overlap of the two ovals. \((A \text{ and } B')\) is the black area of the oval on the left. These areas together make up \(A\).

There is no overlap between \((A \text{ and } B)\) and \((A \text{ and } B')\). Accordingly, the overlap between these areas is \(\emptyset\).
A = (A and B) or (A and B') from above

\[ p(A \mid S) = p((A \text{ and } B) \text{ or } (A \text{ and } B') \mid S) \]

\[ p(A \mid S) = p(A \mid B \mid S) + p(A \mid B' \mid S) \text{ by the Expansion Theorem} \]

\[ p(A \mid S) = p(A, B \mid S) + p(A, B' \mid S) \text{ change of notation} \]

10.22 Let A and B be any two events. Assume that A is probabilistically independent of B:

\[ p(A \mid B, S) = p(A \mid S) \]

Prove that B is probabilistically independent of A.

\[ p(B \mid A, S) = p(B \mid S) \]

The proof is done by using the definition of conditional probabilities twice, once for the joint event A,B, and once for B,A.

\[ p(A, B \mid S) = p(A \mid B, S) p(B \mid S) \]

\[ p(A, B \mid S) = p(A \mid S) p(B \mid S) \]

\[ p(A, B \mid S) = p(B \mid A, S) p(A \mid S) \]

\[ p(B \mid S) = p(B \mid A, S) \]

10.23 Conditional probability is defined as:

\[ p(A \mid B, S) = \frac{p(A, B \mid S)}{p(B \mid S)} \]

Show that the definition of conditional probability satisfies the three axioms of probability introduced in Equations 10-10, 10-11, and 10-12.

1. Probabilities are numbers between zero and one. We start with the definition of conditional probability.

\[ p(B \mid S) \]

Both factors on the right are probabilities and therefore are non-negative.

\[ p(A, B \mid S) \geq 0 \]

\[ p(B \mid S) \geq 0 \]

Therefore the conditional probability is non-negative.
ANSWERS TO PROBLEMS AND DISCUSSION NOTES

$p(A|B,S) \geq 0$

Note that $p(A,B|S) = 0$ when $p(B|S) = 0$, making $p(A|B,S) = 1$ in that case, rather than infinity.

2. Probabilities sum to one. Collectively exhaustive and mutually exclusive means that we are summing over the entire sample space $I$.

$$p(I|B,S) = p(I,B|S) / p(B|S)$$

$I,B = B$

$$p(I|B,S) = p(B|S) / p(B|S)$$

$p(I|B,S) = 1$

3. If $(A \text{ and } B) = \emptyset$, then $p(A \text{ or } B|S) = p(A|S) + p(B|S)$. We introduce the event $C$ for use in the expansion.

$$p(A \text{ or } C|B,S) = p((A \text{ or } C),B|S) / p(B|S)$$

$$p(A \text{ or } C|B,S) = p((A \text{ and } B) \text{ or } (C \text{ and } B)|S) / p(B|S)$$

$$p(A \text{ or } C|B,S) = [p(A \text{ and } B|S) + p(C \text{ and } B|S)] / p(B|S)$$

$$p(A \text{ or } C|B,S) = p(A,B|S) / p(B|S) + p(C,B|S) / p(B|S)$$

$$p(A \text{ or } C|B,S) = p(A|B,S) + p(C|B,S)$$

This shows that the third axiom applies to conditional probabilities.

10.24 Suppose that the moment technique described at the end of Chapter 7 has been used to evaluate a business portfolio. The mean of the distribution is $230$ million, the variance is 18,212 in units of $(\text{million})^2$, and the skewness is 5,000,000 in units of $(\text{million})^3$.

a. Large numbers like this are difficult to interpret. More convenient are the standard deviation, $s$, and the skewness coefficient, skewness$/s^3$. A skewness coefficient outside the range -2 to 2 means the distribution is quite skewed. What is the standard deviation and skewness coefficient for this portfolio?

The two parameters are calculated as

$$\sigma = \sqrt{18,212} = 135$$

$$\text{skewness coefficient} = \frac{5,000,000}{135^3} = \frac{5,000,000}{2,460,375} = 2.03$$
The value of the skewness coefficient, 2, shows that the distribution is quite skewed.

b. Decision-makers relate better to graphics than to numbers like variance and standard deviation. Use the normal distribution (Equation 10–50) to plot a cumulative probability distribution that has the given mean and variance.

The plot is shown after section c below.

c. Creating a plot that matches mean, variance, and skewness is more difficult. Use the shifted lognormal distribution

\[ \frac{1}{\sqrt{2\pi s(x-x_0)}} e^{-\frac{1}{2}\left(\ln\left(\frac{x-x_0}{s}\right) - m\right)^2} \]

to plot a cumulative probability distribution that matches the given mean, variance, and skewness. (The more familiar, non-shifted lognormal distribution has $x_0 = 0$.)

A lot of algebra was required to obtain the equations shown in the hint. The results can be quite useful if you ever need to do calculations with moments. Using the equations, we find:

- $m = 5.253$
- $s = 0.558$
- $x_0 = 8.254$

Using the NORMDIST and LOGNORMDIST functions of Microsoft Excel, we can generate the following graph.
d. Compare the results of b and c. How different are the distributions? How would the plot differ if skewness were 50,000,000? 500,000?

In generating the results for c, you may have noted that the values at the lower end of the graph (mean – 2s) could not be calculated for the lognormal. This illustrates the problems of trying to find a curve that is a good general curve to represent a probability distribution, matching more than the first two moments.

Having gone through all the effort to set up the spreadsheet, it is easy to do the graphs for different values of skewness.

As you can see, the curve shifts dramatically for the larger value of skewness. The tail of the distribution off to the right of the plot becomes more important in preserving the first two moments. Note that the skewness coefficients plotted above are .2, 2, and 20. The usefulness of the lognormal representation of the distribution for values of the skewness coefficient much over 2 is suspect.

*Hint: Use Microsoft Excel or another spreadsheet program to create the graph. Plot the distribution from (mean – 2s) to (mean + 2s).*

For the lognormal distribution with \( x_0 = 0 \), \( m \) and \( s \) are the mean and standard deviation of \( \ln(x) \). The moments of \( x \) are given by

\[
\mu_i = e^{m + \frac{1}{2}s^2}
\]
The following calculations can be used to obtain \( x_0 \), \( m \), and \( s \) from the moments \( m_1 \) (mean), \( m_2 \) (variance), and \( m_3 \) (skewness). All that is required to obtain these results is some tedious algebra and solving a cubic equation to determine \( d \). First calculate

\[
\mu_k = \mu_1^k e^{\frac{k(k-1)s^2}{2}} = e^{km_1 + \frac{1}{2}m_3s^2}
\]

These values can then be used to calculate the parameters we need:

\[
w = 2v_2^3 / v_3^2
\]

\[
r = \sqrt{1 + 2w}
\]

\[
d = \frac{v_2^2}{v_3} + v_2 \left( \frac{(1 + w + r)^{1/2} + (1 + w - r)^{1/3}}{2v_3^{1/3}} \right)
\]

\[
A = v_2 + d^2
\]

\[
B = v_3 + 3dv_2 + d^3
\]

Note that if the skewness is negative, you need to use the reflected shifted lognormal. To do this, use the absolute value of the mean \( m_1 \) and skewness \( m_3 \) in the above calculations, and then substitute \((x_0 - x)\) for \((x - x_0)\) in the equation for the shifted lognormal above.
Problems and Discussion Topics

11.1 How does an influence diagram contribute to making good decisions? (Refer to the elements of a good decision.) What elements of a good decision does an influence diagram not help with and why?

An influence diagram helps you develop the decision basis. It starts with the values and proceeds to develop in an easy and logical manner an understanding of the alternatives and information (principally uncertainties) that are important to the decision. It develops the logic of the problem explicitly and can function as a calculation tool to arrive at the answer implicit in the decision basis. Finally, it records the input and the process and provides a basis for judging the quality of the decision.

The influence diagram does not play an important role in two steps of the decision analysis process. The first step is alternative generation. This step requires creativity amid complexity, and the strategy table is the best device we know for making sense out of chaos during the alternative generation phase. The second step in which the influence diagram does not play a large role is creating the deterministic model. However, there is an important relationship between the two: the model must be able to deal with the variables in the influence diagram. But it must also deal with many factors and relationships that do not appear explicitly in the influence diagram.

11.2 Describe at least one way that using influence diagrams helps you draw better decision trees and one way that being familiar with decision trees helps you draw better influence diagrams.

Students may come up with many answers to this question. Below are some suggestions.
Influence diagrams capture complex problems that initially would be very difficult to represent in a decision tree. After the deterministic sensitivity analysis simplifies the influence diagram, a decision tree that truly and efficiently represents the process can be drawn. In addition, the influence diagram keeps track of probabilistic dependency, a factor that is all too easily neglected or forgotten in a large decision tree.

Decision trees, on the other hand, are more natural means for representing “asymmetric” situations in which one branch of the tree can lead to a different set of nodes than another branch. This type of information is difficult to represent intuitively and efficiently in influence diagram form. It is often better to work out the logic in simple tree form and then use this structure to construct an efficient influence diagram.

11.3 How do you know when an influence diagram has become complicated enough? Relate your answer to the problem of assessing probabilities and to the clairvoyance test.

In principle, an influence diagram is complicated enough when the addition of another node does not help you in assessing probabilities or in understanding the structure of the problem. In practice, the people developing the influence diagram easily recognize this point of diminishing returns.

When the influence diagram is very simple, the uncertainties tend to be very aggregate, and the assessment of probability is very difficult. People want to break the assessment into different cases—“If we assume competition reacts strongly, then the probabilities are...” ; this usually indicates that a new node (e.g., Competitive Reaction) should be added to represent the possible cases. On the other hand, when the influence diagram has grown too detailed, people cannot use their intuition effectively and will instinctively move to the level at which they are comfortable—“I can’t think of the reaction of Competitor A and Competitor B separately, so let me just think of the combined competitive reaction.”

The situation is somewhat similar to the clairvoyance test: if the person being assessed asks questions before giving a probability, it is not well enough defined. Similarly, with influence diagrams, if the person being assessed begins to break the assessment into different cases, you are at too great a level of aggregation. In both cases, better definition of the question is needed.

11.4 Think of a significant decision you have made. Draw the influence diagram for that decision. Were there significant uncertainties? How did you identify and deal with them at the time? Do you have any new insights into the decision? (Relate this last answer to the good decision/good outcome distinction.)

This is a question designed to make students relate the decision analysis process to their lives. Sometimes in personal decision-making it is difficult to separate decisions, uncertainties, and values. The answers to this question should, at the very least, identify these three elements clearly. The answer should also distinguish the quality of the decision from the desirability of the
outcome that occurred. The student might also note that the full outcome of significant personal decisions may take many years to unfold, and he or she may not yet be able to fully characterize the outcome as good or bad.

11.5 Draw the influence diagram for the date and time when a specific close relative walks through your front door. Make sure the uncertainty passes the clairvoyance test and try to summon all your information and experience on the factors influencing the uncertainty. Has the exercise changed your understanding of the uncertainty at all? Could you now draw a decision tree and do a meaningful probability assessment? Draw the tree and explain what (if anything) would prevent you from assessing probabilities and calculating the expected date and time that the relative walks through your door.

This question is designed to make students realize the amount of information they possess. The influence diagram will help them break the problem down to the level at which intuition works best, use the information, and combine the results to obtain the probability distribution on date and time.

An interesting facet to this analysis could be the inclusion of such decisions as do not contact, but just wait for the relative to show up; phone and invite the relative to a party; write a letter requesting money; etc.

11.6 Draw an influence diagram for the probability of a major war within the next ten years. Make sure your uncertainties pass the clairvoyance test. How is this problem different from the previous one? Is there anything preventing you from drawing a tree and calculating a probability distribution for this problem?

One of the difficult parts of this question is finding a definition of “major war” that comes close to passing the clairvoyance test. Possible elements of the definition are the number of soldiers deployed outside the country, the number of casualties per month, and the level of weapons used.

This problem differs from problem 11.5 in two aspects. First, in that problem, only one or two people share the state of knowledge upon which the probability judgments are based; in this problem, most of the uncertainties are open to public debate and conflicting estimates, and the problem is less one of communicating one’s state of knowledge and more of reconciling (if possible) several states of information. Second, most students are not confronted with decision alternatives that affect the probability of the outcome.

11.7 Draw the influence diagram for the number of times you eat pizza within the next month. Again, make sure the uncertainty passes the clairvoyance test. Are there any difficulties in completing this problem and, if so, what are they?

The novel factor in this problem is the difficulty in distinguishing between decisions and uncertainties. If there were a sufficiently compelling reason, you could decide exactly how often you will eat pizza within the next month and then follow through on that decision. Is this the problem you wish to analyze? Or is the problem more like “Given the uncertain state of my
finances and social life, how often will I eat pizza next month?" Make sure the clairvoyant would not be confused by your definition of the problem!

A similar problem occurs for small companies when they realize that they can determine exactly how much of a product they will sell next month—all they have to do is adjust the price and supply correctly! The real decision, though, is probably not how much to sell, but how to maintain current profits and enhance the value of ongoing business.

11.8 In the influence diagram used to construct the tree in Figure 11-11, there is an arrow pointing to the left.

a. Reverse this arrow to make the diagram a “decision tree network,” one in which the nodes can be arranged so that all the arrows point to the right.

The relevant nodes of the influence diagram in Figure 11-11 are the three nodes shown below.

```
 Raw Material
 Supply Strategy

 Survey Results

 Raw Material Cost, 1997
```

We can use the procedures for manipulating influence diagrams to reverse the left-pointing arrow between the Survey Results node and the Raw Material Cost, 1997 node. First, we need to add an arrow between the Raw Material Supply Survey node and the Raw Material Cost, 1997 node, which makes the two chance nodes share the same state of information—a prerequisite for arrow reversal.

```
 Raw Material Supply Strategy

 Survey Results

 Raw Material Cost, 1997
```

The arrow between the two chance nodes can now be reversed, as shown below. This makes the influence diagram a decision tree network.

```
 Raw Material Supply Strategy

 Survey Results

 Raw Material Cost, 1997
```

b. Why is it necessary to reverse this diagram to create a tree? (Hint: How would you display the probabilities at node 2 in the tree?)

In Figure 11-11, the two uncertainty nodes are in the order shown below; the four intervening nodes have been omitted to keep the tree drawing manageable. In this order, it is not possible to show the probabilities for the first node since they will be different for each branch of the second node. This problem of representation does not occur for the decision tree network represented by the influence diagram above. Note that the process of manipulating the
influence diagram accomplishes the same task as using Bayes’ Rule to recalculate the probabilities for use in the tree; in fact, it is just these recalculated probabilities that are contained in the influence diagram with the reversed arrow.

11.9 Adding an arrow between an uncertainty and a decision node is related to the value of information calculation described in chapters 2 and 4.

a. Draw the trees represented by influence diagrams A and B above. Assuming the structure of the data used in Chapter 11 of the textbook, we can draw the trees shown below.
b. How are influence diagrams A and B related to the value of perfect information on Market Growth Rate?

In diagram A, price is decided before the market growth rate is known; in diagram B, the market growth rate information is available before price is decided. The difference between the expected value (certain equivalent) of these two diagrams is the value of perfect information on market growth rate.

11.10 In the Howard canonical form of an influence diagram, there are no arrows from a decision node to any uncertainty node aside from the value node. For the purposes of this definition, groups of uncertainty nodes can be amalgamated into a larger uncertainty node (the value node) provided no loops are created.

A company has several different routes it could pursue in developing a new product. The influence diagram representing its problem is shown below.
a. *Is the influence diagram in Howard canonical form? The one below?*

The first diagram has an arrow from the Product Development decision node to the Development Results uncertainty node. There is also an arrow from the Development Results node to the Product Introduction node, and, for this reason, the Development Results node cannot be amalgamated into the NPV node without creating a loop. Therefore, the diagram is not in Howard canonical form. The second diagram is in Howard canonical form. (Note that the node Development Results Learned is a deterministic node rather than an uncertainty node.)

Insofar as possible, the Howard canonical form avoids situations in which decision nodes influence uncertainty nodes. In many cases, uncertainties like the Development Results node are characteristic of what we know about the possible product; even if we do not decide to develop the product, it has the same characteristics!

b. *Suppose some preliminary work could predict the results of the product development effort before all the necessary development work was done. Which influence diagram could be used to calculate the value of information about development results?*

The first diagram cannot be changed to calculate the value of information on Development Results; this would involve adding an arrow from the Development Results node to the Product Development node, and there is already an arrow from Product Development to Development Results. The existence of the two arrows pointed in opposite directions would create a loop, and this is not allowed in influence diagrams.

There is no problem of this type in the second influence diagram. An arrow from Development Results to Product Development can be added without creating a loop, and the resultant diagram represents the problem with perfect information on Development Results.

c. *Which influence diagram is in Howard canonical form? Draw a tree showing the logic contained in the deterministic node that would make
The second influence diagram above is in Howard canonical form because there are no arrows between decision nodes and uncertainty nodes other than the value node. The deterministic node contains the logic shown in the distribution tree below, assuming development can lead to either success or failure. Note that there are no circles or squares in the distribution tree for a deterministic node; it represents only logic and does not describe probabilities or alternatives.

\[
\begin{array}{c}
\text{Development} \\
\text{Yes} \\
\text{No}
\end{array}
\quad
\begin{array}{c}
\text{Development Results} \\
\text{Success} \\
\text{Failure}
\end{array}
\quad
\begin{array}{c}
\text{Product Development} \\
\text{No Results}
\end{array}
\]

\[
\text{Development}
\quad
\begin{array}{c}
\text{Success} \\
\text{Failure}
\end{array}
\quad
\text{Product Development}
\quad
\begin{array}{c}
\text{No Results}
\end{array}
\]

\[\text{Yes} \quad \text{No} \quad \text{No Results}\]

\[\text{Development Results} \quad \text{Success} \quad \text{Failure}\]

\[\text{Product Development} \quad \text{No Results}\]

**d. Can the second influence diagram be manipulated to a form from which the value of information about development results can be calculated?**

No, the second influence diagram cannot be manipulated to a form from which the value of information about development results can be calculated. The procedure for manipulating an influence diagram does not change the meaning of the influence diagram, and we need to change the information in the influence diagram to take into account the value of information.

You need to modify, not manipulate, the diagram in order to change its meaning and calculate the value of information.

Adding an arrow from the Development Results node to the Product Development node is the step needed to create the new influence diagram describing the situation in which information is obtained before the decision is made; the expected value (certain equivalent) of this new diagram minus the expected value (certain equivalent) of the original diagram is the value of perfect information.

The arrow added in this process is between an uncertainty node and a decision node, and adding this arrow is not one of the steps allowed for manipulating influence diagrams; you can add arrows only between uncertainties.

\[11.11 \text{ It is possible to use deterministic nodes to represent asymmetries in the problem in a straightforward way.}\]
a. Redraw the influence diagram in Figure 11-11 using the deterministic node defined above. What arrows should be drawn from these nodes to the remainder of the diagram?

The first three nodes of the influence diagram in Figure 11-11 are replaced with the four nodes shown below. The arrow to the Survey Results node is the same arrow as in Figure 11-11—an arrow from Raw Material Cost, 1997. All the arrows from the Pricing node to the remainder of the diagram are unchanged.

b. What is the logic contained in the deterministic node Survey Results Learned?

The logic in the deterministic node is shown in the distribution tree below. Note that there are no circles or squares in the distribution tree for a deterministic node; it represents only logic and does not describe probabilities or alternatives.
c. Is the tree drawn from the new diagram different from the tree in Figure 11-11?

No, the tree drawn from the new diagram is identical to the tree drawn in Figure 11-11.

d. Make this diagram into a decision tree network. (See problem 11.8 for the definition of a decision free network.)

As in problem 11.8, the task is to reverse the arrow between the nodes Survey Results and Raw Material Cost, 1997. The diagram below contains all the relevant nodes from Figure 11-11.

There is no arrow leading into either node, so the arrow can be reversed without changing anything else in the influence diagram.
e. Is the influence diagram in Figure 11-11 in Howard canonical form? (See problem 11.10.)

There is an arrow from the Raw Material Supply Survey decision node to the Survey Results uncertainty node. There is also an arrow from the Survey Results node to the Pricing Strategy decision node; therefore, the Survey Results node cannot be amalgamated into the value node. The influence diagram thus is not in Howard canonical form.

f. Is the influence diagram drawn as part of this problem in Howard canonical form?

In this influence diagram, there are no arrows from decision nodes to uncertainty nodes that cannot be amalgamated into the value node without creating a loop. Therefore, it is in Howard canonical form.

11.12 In Chapter 10, a joint probability distribution was given for the two sets of mutually exclusive and collectively exhaustive events, \( R_i \) and \( C_j \).

\[
p(R_i, C_j | S) \\
\begin{array}{ccc}
  | & C_1 & C_2 & C_3 \\
  R_1 & .10 & .25 & .03 \\
  R_2 & .22 & .26 & .14 \\
\end{array}
\]

A. \( R, C \)  
B. \( R \)  
C. \( C \)  
D. \( R \rightarrow C \)  
E. \( C \rightarrow R \)
Draw the distribution trees for the nodes in the five influence diagrams above.

A: The distribution tree is on the joint events.

B: This distribution tree is for the marginal probabilities for R.

C: This distribution tree is for the marginal probabilities for C.

D: There are two distribution trees in this influence diagram: the marginal probabilities for R and the conditional probabilities for C.
E: There are two distribution trees in this influence diagram: the marginal probabilities for C and the conditional probabilities for R.
11.13 In Figure 11-1, there is no arrow going to the Market Growth Rate node. The lack of arrows is of great significance to the analyst, since the absence of arrows makes modeling and probability assessment relatively simple.

a. There is no arrow between the nodes Pricing Strategy and Market Growth Rate. What does this indicate about the nature of the market and Medequip’s place in the market? Under what circumstances should an arrow be drawn between these two nodes?

The lack of arrow indicates that Medequip is not a dominant player in the market. Whether it competes in targeted segments of the market at relatively high prices or whether it competes in all segments of the market has no great effect on the uncertainty in the size of the market. If Medequip dominated the market, on the other hand, or if Medequip’s technology had unique and important characteristics, then its pricing strategy could well affect the uncertainty on how rapidly the market grew.

b. There is no arrow between the nodes Competitor’s Reaction and Market Growth Rate. What does this indicate about the nature of the market and the companies that supply products in this marketplace? Under what circumstances should an arrow be drawn between these two nodes?

The lack of arrow indicates that the size of the market is not sensitive to the actions of Medequip and its competitor. Whether prices are high or low (price war), the uncertainty on market size will be the same.

c. Under what circumstances should there be an arrow from both the Pricing Strategy and Competitor’s Reaction nodes to the Market Growth Rate node?

If the uncertainty for market size is price sensitive, there should be an arrow from Competitor’s Reaction to Market Growth Rate. If Medequip’s technology is especially important to different segments of the market, there should also be an arrow from Pricing Strategy to Market Growth Rate.

d. We are not allowed to draw an arrow from Pricing Strategy to Competitor’s Reaction, from Competitor’s Reaction to Market Growth Rate, and from Market Growth Rate to Pricing Strategy (perhaps to represent a pricing adjustment to changes in market dynamics). Why is this not allowed? How might you represent an adjustment of pricing strategy to market dynamics?

These arrows would create a loop, which is not allowed in influence diagrams. To represent an adjustment of pricing strategy to market dynamics, we could introduce a second Pricing Decision to represent a reconsideration of pricing some time later. To represent this, introduce the decision node Repricing Decision. We could then have an arrow from Market Growth Rate or from Competitor’s Reaction to the node Repricing Decision to adjust pricing strategy to the market response to the first Pricing Decision and the subsequent Competitor Reaction.
12

Encoding a Probability Distribution

Problems and Discussion Topics

12.1 Discuss the differences in the processes by which motivational and cognitive biases arise. What implications do these differences have for the methods to overcome them?

The basic point to be emphasized is that motivational biases arise when the subject wishes to tailor his or her response to produce a particular result, while cognitive biases arise when the subject incorrectly uses his or her information. Accordingly, motivational biases can be overcome by decoupling probability information from whatever process the subject is trying to influence. For instance, this could mean setting the subject’s performance goals separately from capturing the subject’s state of information. On the other hand, cognitive biases can often be overcome through exercises that help the subject to better process the information he possesses.

12.2 List a bias that commonly arises in an area other than probability assessment. How is the bias recognized and overcome? If it is not overcome, is it because it is not possible to do so or not important enough to do so? Or is it because it is not even recognized?

Biases, or tendencies to look at or evaluate things in a particular way, are probably more the rule than the exception. They range from a father seeing only his daughter on stage at the dance recital to the incumbent’s assessment of her administration when she is running for reelection. Although these biases are often recognized, they are likely to be overcome only when there is a strong incentive to do so. One particular area where there are strong incentives to overcome biases are in evaluating oil drilling sites; systematic bias can lead to millions of dollars in unproductive drilling costs or lost profits.
12.3 You are about to assess the probability distribution on the average growth rate over the next year for the entire energy industry. The expert is a market analyst who closely follows the stocks of the large oil companies. What biases might you expect to encounter?

Motivational biases are less likely to be present because the analyst is probably not compensated according to the analyst’s prediction of the growth in the energy industry. However, a number of cognitive biases are possible. The expert may focus too much on changes in this week’s stock prices and neglect the prospects for the industry over the next year (representativeness bias). Or the expert may give you a forecast very close to this year’s actual growth rate because that information is in the forefront of consciousness (adjustment and anchoring). If part of the expert’s job is to generate one or two different scenarios for the coming years, he or she may feel that one scenario is more plausible than the other and focus unduly on the growth rate in that scenario (implicit conditioning bias).

12.4 One technique for overcoming several kinds of biases is called the “Rip Van Winkle Technique.” To apply it, you would discuss with the subject the highest and lowest possible outcomes of an uncertain variable. You would then say that it is a number of years after the actual outcome of the variable was discovered. The two of you run into each other again. You inform him that the variable turned out to be 10 percent higher than his highest possible estimate years before. You ask him to explain how it turned out higher than either of you had thought possible.

Why does this technique work?

The need to explain an outcome higher than a previous extreme estimate requires that people examine their knowledge and draw new inferences from it, rather than relying on previous estimates. This process of reexamination and reevaluation forces people to set aside past conclusions and ponder a broad range of possibilities, allowing information on those possibilities to be better captured in a probability distribution.

12.5 Find a friend to serve as a subject in the following subjective probability experiment. Alternatively, try the experiment on yourself.

a. Tell the subject that a fair coin \( p(\text{head} \mid S) = .5; p(\text{tail} \mid S) = .5 \) will be flipped six times. Assess the subject’s cumulative probability distribution on the number of heads that occur in the six flips. Discourage your subject from trying to make any mathematical calculations of the odds. If you ask the questions in the right way, it will be very difficult for him or her to make any such calculations.

The trick here is to pose the questions in the less-than or greater-than form of the cumulative or excess probability distributions; e.g., would you rather
b. Now tell your friend that you have three coins (two of which are unfair) with different probability distributions.

1. \( p(\text{head} \mid S) = .25, \quad p(\text{tail} \mid S) = .75 \)
2. \( p(\text{head} \mid S) = .50, \quad p(\text{tail} \mid S) = .50 \)
3. \( p(\text{head} \mid S) = .75, \quad p(\text{tail} \mid S) = .25 \)

Then tell the subject that one of those coins will be randomly selected and flipped six times. Assess his or her subjective cumulative distribution on the total number of heads that result.

c. Calculate the actual distributions for a and b under the given assumptions. Compare these distributions with the assessments from your subject. Also note any difference between the subject’s distributions in a and b. What might explain the differences, if any, between the various distributions?

Below is the calculated distribution for a, with an expected value of 3.
Below is the calculated distribution for b, with an expected value of 3.

As you can see, the expected values are the same, but the distribution for a is steeper in the center than for b. With b, there is more uncertainty about how many heads will result because of the additional uncertainty about whether the coin used will have a .25, .50, or .75 probability of heads. If the distributions you assessed for a and b do not show similar characteristics, then your subject may have been subject to a bias. The most likely possibility is anchoring to the first distribution and failing to incorporate the new information and additional uncertainty for the second one.

12.6 Break into groups of two or three and encode probability distributions. Role playing by the “expert” and the “analyst” can help make the exercise more realistic, especially if assumed motivational biases are written down beforehand (but not revealed to the analyst). The quantity encoded should be a continuous variable for which the uncertainty will be resolved sometime after the encoding session. Be sure to spend time describing and structuring the variable and exploring the possibility of biases. You may find it useful to structure a simple influence diagram with the subject before assessing the probability.

Some possible topics for assessment are (make sure the definitions pass the clairvoyance test):

a. The price of a stock two weeks from now
b. The difference in temperature between Stockholm and Rio de Janeiro on
a particular day

c. The number of people attending a large undergraduate class on a given
day.

An assessment tool (such as a probability wheel) would aid greatly in this
exercise, as well as in the other assessment exercises. Cumulative probabil-
ity paper also helps. (If probability wheels are not available locally, laminated
cardboard ones can be purchased from Navigant Consulting, Inc. for $9–10
each.)

12.7 Slippery Company produces, among other things, special types of lubricants
for specific mechanical applications. There is one type of lubricant it does not
produce. This lubricant is currently produced by several large companies from
a feed stock of ethylene. Since ethylene prices are rising along with petroleum
prices, the cost to produce this lubricant is rising. (This case is a disguised
version of an analysis done in the late 1970s.) Slippery knows that the
lubricant can be made from the oil of the “oily bean” at a cost that appears
competitive today with the ethylene-based process. Since oily bean oil prices
are not rising, Slippery is considering constructing a facility to produce the
lubricant from oily bean oil. However, two factors worry Slippery. First, there
is a rumor that several other companies are considering the same move, which
would saturate the market with cheap lubricant. Second, although oily bean oil
prices are fairly constant, droughts make the price jump temporarily every
couple of years.

a. Structure the problem and determine your information needs. Be sure to
draw an influence diagram.
b. *For one of the necessary items of information, designate an expert, motivate the expert, and assess a probability distribution on the item.*

Probability distributions could be assessed on any of the above uncertainties, including different lubricant price distributions for different ethylene price/competitor scenarios. Or, if desired, some of the following information could be given to the student to facilitate full tree analysis and an assessment assigned on the missing information:

- Ethylene price of $0.30/lb. with possible real (constant dollar) growth rates of 2 percent, 6 percent, and 10 percent per year and corresponding probabilities of .25, .50, and .25. Two pounds of ethylene are needed per pound of lubricant produced. Other costs amount to $0.30/lb of lubricant, leaving an operating margin of $0.10/lb of lubricant.
- Oily bean oil price of $0.20/lb. with the price constant in real dollars and two pounds required to produce a pound of lubricant. On the average, every eight years the price of oily bean oil doubles for a year and then returns to its original price.
- Plant construction cost of $10 million, with $500,000 in fixed costs per year and a production capacity of 10 million pounds of oily bean oil per year.

Competitors will enter the market within 6 to 10 years, resulting in a market glut that drops the operating margin on the lubricant to only $0.01/lb of lubricant.